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An alternative pricing model for options on wheat futures with time varying deterministic volatility*

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Abstract

Substantial differences in implied volatilities of wheat options traded in Chicago and Paris are observed, which have important valuation implications for wheat crop insurance contracts. This study aims to analyse and explain the observed differences along various dimensions. It discusses differences in historical volatilities, the market structure, and their consequences for a valuation model. These findings, combined with in-depth analysis of seasonality and scarcity-related factors are used to suggest a time varying deterministic volatility model for wheat futures. This volatility model is used to provide a pricing model for wheat options in a risk-neutral measure.

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1 Executive summary

1.1 Initial situation

A typical wheat crop insurance contract under this study is valued in autumn, i.e. before sowing, and expires after or around harvesting time. Once the crop insurance contract is underwritten, the insurer keeps the risk over the contract's lifetime, which is roughly one year. Implied volatilities of exchange traded options on wheat futures are important factors influencing the value of a crop insurance contract. In general, a higher implied volatility demands for a higher crop insurance premium to cover the inherent wheat price risk. Thus, a profound understanding of implied volatilities and its driving factors are necessary conditions to determine an adequate insurance premium.

A question of important practical relevance is the observed differences in implied volatilities among exchanges, in particular between Chicago and Paris. Taking the 07. February 2018 as example, the implied volatility of a traded wheat option in Paris, with an expiration in September, is roughly 60% of a comparable option traded in Chicago, which implies an approximately 40% lower option price in Paris.[1] Explaining these observed differences combined with a suggestion for a pricing model for wheat options would improve the process to determine the price of a wheat crop insurance contract.

1.2 Scope of the study

This study derives a pricing model for exchange traded wheat options with a strong focus on a pricing model that can easily be applied in daily business. An exchange traded commodity option is a derivative asset whose value depends on the characteristics of its underlying. The underlying, however, is – with almost no exception – a derivative itself, namely a commodity futures, rather than (the spot price) of a physical good. Neglecting this fact in the valuation of the respective option, as is typically done in the existing literature, unnecessarily increases both model complexity and the risk of misspecification or estimation errors. Thus, special emphasis is placed on the unique characteristics of (i) the option's underlying, (ii) the market for such options, and (iii) the wheat market in general, all of which have an important influence on the option's price and therewith on its implied volatility.

Another focus is set on the analysis of the seasonal pattern of the underling's volatility during a marketing year as well as the time to maturity effect, which postulates an increasing volatility as a futures approaches maturity[2]. The study further addresses the impact of storage on the characteristics of the underling's volatility. Because storage plays a crucial role in the context of price fluctuations, since it is a buffer against supply and demand shocks.

[1]For this example, an at the money call option is used with 20% volatility in Chicago and 12% in Paris, a duration of 7 months and an interest rate of 1.5% a year using the standard Black-Scholes option pricing model.

[2]The time to matuity effect is commonly known as the Samuelson-effect, firstly mentioned by Samuelson (1965).

1.3 Market structure

To provide a profound basis for modelling the underling's volatility, the study discusses the specific characteristics of major commodity exchanges, as well as the differences in the contract specification of the option's underlying, i.e. wheat futures, traded in these markets. In particular, the relevance of four factors, namely: currency, quality/grading standards[3], market integration and trading liquidity is analysed. In order to account for quality/grading standards, the market sample is enlarged by the exchanges Kansas and Minneapolis.

Findings presented in the study suggest that the EUR/USD exchange rate is of primary importance in explaining volatility differences between wheat futures traded in Paris and the U.S. In particular, the inverse relationship between the USD value and commodity prices enlarges volatility of contracts valued in USD. Second, quality differences – which determine the substitutability between wheat varieties – contribute substantially to explaining differences in volatility. Lower variance differences are observed when the quality spread between contracts narrows, which is the case for Paris and Minneapolis. Third, trade policy measures, such as import quota, taxes or tariffs, which determine the degree of market integration explain parts of the time varying behaviour of volatility spreads. As an example, the volatility spread between Paris and all U.S. wheat contracts has significantly narrowed after 2005 with the reduction of EU import tariffs on wheat. Liquidity differences, in contrast, appear of minor importance in explaining volatility differences.

1.4 Seasonality and storage

The second part of this study focuses on the seasonal behaviour of the option's true underlying: a futures on wheat. This is a convenient feature, since analysing and interpreting the implicit structure of implied volatility of options directly would be more complex, due to the smoothing effect of time to maturity. The findings can be summarised as follows: first, seasonality and time to maturity are important elements for understanding and describing volatility of wheat futures during a marketing year. Second, the introduced volatility model provides a simple way to parametrize the seasonality and time to maturity effect and helps to disentangle both effects. In particular, higher volatility is found around harvest and lower during the winter season.

A valuable outcome of this study is the simple way to model the impact of storage on the underling's price volatility. Because, storage, as a fundamental driver of volatility and seasonality, provides a profound basis for modelling (rare) extreme wheat price volatility. A suitable proxy for storage is the U.S. stocks-to-use ratio, measured and published by the U.S. Department of Agriculture on a monthly basis. Empirical findings show that stocks-to-use ratios have predicting power for modelling (rare) extreme wheat price volatility and suggest large volatility spikes around harvest when stocks are low.

[3]In this study, quality and grading standards are referred as to the exchange specifications for physical delivery, see for example CME rule book, chapter 7 "Delivery facilities and procedure for agricultural commodities and ethanol."

1.5 Option pricing model

Finally, the study builds on the seminal Black (1976) model and suggests an extension by considering the most relevant volatility characteristics of exchange traded wheat futures. The model has three desirable properties: first, its simple parametrization is a key feature for practical implementations and makes the model less vulnerable to over-parametrization. Second, it simplifies the valuation impact of extreme volatility shocks. Third, the model bridges the gap between observed futures volatility and implied volatility, i.e. the market expectation about the volatility over the remaining marketing year. Comparing volatility forecast indicates that in less liquid option markets the introduced volatility model provides substantial gain, while implied volatility appears to be an adequate choice in highly liquid option markets.

1.6 Structure of the study

The study is organised into seven parts:

Section 2 provides a market overview and describes the characteristics of important commodity exchanges on which wheat futures and options are traded. It starts with a brief review on the fundamentals of wheat followed by a description of possible relevant factors driving volatility differences between these exchanges.

Section 3 focuses on the impact of four factors, namely: currency, quality/grading standards, market integration and trading liquidity, which potentially explain volatility differences of wheat futures traded among various exchanges.

Section 4 provides empirical evidence for seasonality of wheat futures' volatility and the time to maturity effect. These findings are the first core of this study and are used to introduce a deterministically time varying volatility model.

Section 5 presents a theoretical foundation on how inventory levels may affect the volatility of commodity prices and under which circumstances large volatility spikes are likely to occur. Subsequently, this theory is empirically tested using the U.S. stocks-to-use ratio.

Section 6 is the second core of this study, where an option pricing model is developed. It incorporates the main characteristics of wheat futures as described in section 4 and can be easily applied in daily business.

Section 7 compares realised volatility of September wheat futures with the predictions of the introduced volatility model and with the market expectations, i.e. implied volatility, over the contract's lifetime.

Section 8 concludes the study.

2 Market overview

This section focuses on the characteristics of important commodity exchanges on which wheat futures and options are traded and starts with a brief review on the fundamentals of wheat, including production, consumption and storage. The importance of such characteristics are often underestimated in the academic literature, although substantial distinction can be made between kinds of wheat traded at various commodity exchanges. This characteristics are important drivers of price and volatility differences.

2.1 Fundamentals of Wheat

Wheat is the staple food of mankind and the most important grain for human consumption but also used to feed animals.[4] Compared to other grains, wheat provides more food energy to humans than any other crop and is today still the staple diet for many nations.

Kinds of wheat

A large array of wheat varieties exists. The U.S. industry uses six widely accepted classifications, which are based on the growing season and on the protein content. While worldwide many different classifications are used, depending on species and planting area, it is common to classify wheat as either winter or spring wheat.

Winter wheat is planted in the fall, in the U.S. between August and October, but stops growing during the period of dormancy in the winter. In spring, the winter wheat resumes growth and is harvested in the early summer, i.e. between June and July.

Spring wheat is preferred in areas with harsh winters and less snow, since the plants can be damaged during cold periods, in particular when plants are not adequately protected by snow cover. Spring wheat is planted in the U.S. between April and late May, and harvested from August till September. Crop yields of winter wheat are typically higher compared to spring wheat, due to the longer growing period.[5]

The U.S. marketing year starts in June and ends in May, including the harvest season which starts with winter wheat typically in Texas in June and continuous to north through the summer and completes with spring wheat in Minnesota until September.[6]

Another classification refers to the protein content. In the U.S. *soft wheat* and *hard wheat* stands for low and high protein content, respectively. Contents range from 10% in soft wheat to 15% in hard wheat species[7]. In spring wheat, the protein content is usually higher than in winter wheat. The following five U.S. wheat classifications with ascending order of their protein content are:[8]

[4] It is a cereal grain and belongs to the category of grasses, which are cultivated for their grains or seeds. The domestication of wheat began earlier than 6500 B.C. in the area of Mesopotamia, probably agriculture's birthplace, from where cultivation has spread to the rest of the world.

[5] See, Vogel (1999).

[6] See, Vogel (1999) or World Agricultural Supply and Demand Estimates (WASDE), May 10 2018, page 11.

[7] According to the U.S. wheat associates, the average protein content of U.S. SRW is approximately 10%, see USW (2013) or NYT (1981).

[8] See, "Wheat: Background". USDA. Retrieved 2 October 2016.

- soft white wheat (SWW),
- soft red winter wheat (SRW),
- hard red winter wheat (HRW),
- hard red spring wheat (HRS) and
- durum wheat (DUR).

In general, wheat with a higher protein content commands higher prices.[9] A further distinction is made between white and red wheat. In contrast to white wheat, red wheat may need to be bleached before it is used. Thus, white wheat usually commands higher prices than red wheat.

A common assumption is that wheat has a high degree of substitutability between the variety of classes, quality standards and end use of wheat. This explains why wheat prices of all classes are "mainly" driven by the entire production level of wheat and not by the production level of a specific wheat class. However, it is often observed that price differences between wheat classes are time varying[10]. This observation is in line with the anecdotal evidence that substitution between different wheat classes is avoided as far as possible.[11]

Wheat production

According to the USDA, winter wheat in the Northern Hemisphere is the majority of the world's wheat production, while a large amount of spring wheat is produced in the U.S., Canada and Russia. Wheat is planted in the Southern Hemisphere after the harvest of spring within the Northern Hemisphere, which helps to smooth supply year around. According to the USDA, the annual world wheat production increased between 2000 and 2017 by 30% form 583 to 759 million tonnes, while during the same period the worldwide harvested area increased by less than 3% from 215 to 220 million ha. This effect is a result of ongoing real productivity gains. The world wheat crop yields increased between 2000 and 2017 by 27% from 2.7 to 3.45 tonnes per hectare. For illustrative purposes, Figure 1 shows the worldwide areas of wheat production and respective yields.

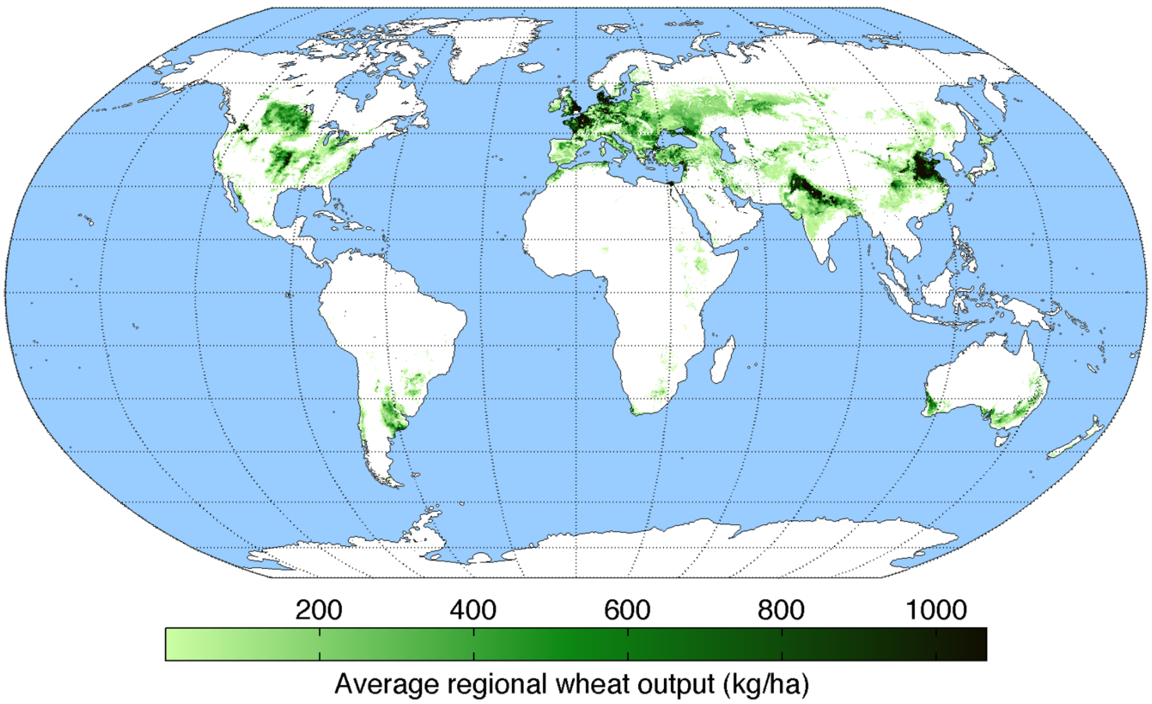
A specific characteristic is that wheat production is dominated by only a few major producing countries. In particular, the five largest producer account for approx. 2/3 of the world wheat production including the European Union (20%), China (17%), India (13%), Russia (11%) and the U.S. (6%).[12] In the U.S., HRW wheat is at most produced with 47% of the total U.S. wheat production, followed by HRS wheat with 21%, SRW with 15%, SWW with 12% and DUR with 5% in 2016.[13]

Wheat consumption

Wheat is primarily used for food, seed and industrial (FSI) as well as for feed consumption. On average, the worldwide fraction of FSI on total domestic consumption is around 80%, with a substantial differences across countries. Wheat used for food consumptions is primarily milled flour to

[9] More details with respect to wheat classes and their value it is referred to Janzen and Adjemian (2017).
[10] See for example Figure 15 in the appendix.
[11] See, Schwager (1997), page 440.
[12] See Bond and Liefert (2018).
[13] See WAOB (2018).

Figure 1: Geographical areas of worldwide wheat yields



Source: FAOSTAT database, "Grain Market Report" 2010.

produce breads, cakes, crackers, pasta, and other edibles.[14] The flour is made thorough a milling process of wheat kernels, and the by product, such as bran, germ, middling and shorts, are used for production of livestock feeds.

In line with production, the amount of wheat consumption increased by the same ratio as production and rose from 584 to 742 million tonnes a year. Consumption is dominated by the EU, China and India, which account for 46% of the worldwide consumption. Remarkably, the largest consumer are also large producers and therewith a large fraction of domestic consumption is not available for international trade.

Global wheat transfer

International wheat trading became more important over the last years. The fraction of wheat used for international trade on total production increased from 17% in 2000 to 24% in 2017. Exporting is dominated by five large producers, namely: Russia (20.6%), the U.S. (13.8%), the EU (13.7%), Canada (12.4%) and Ukraine (9.5%). Together, they account for 70% of total wheat exports. Other large exporters are Australia and Argentina.[15]

Quite different to the concentration of exporters, importing countries are more diverse. The largest five wheat importing countries are: Indonesia, Egypt, Brazil, Algeria and Bangladesh. Together, they account for 25% of total imports and their individual import fraction ranges between

[14]See Dunsby et al. (2008), page 110.
[15]See Bond and Liefert (2018).

6.8% and 3.5%.[16]

Wheat storage

Storage is used to smooth supply over a marketing year. Countries or production areas have different marketing years, depending on the start of harvesting the new crop. For example, in the U.S. the wheat marketing year runs from June through May, while the international marketing year runs from July through June.[17]

In accordance with global wheat consumption, global wheat ending stocks, i.e. amount stored, increased between 2000 and 2017 by 30% from 206 million tonnes to 269 million tonnes.[18] However, stocks are very heterogeneously distributed across the globe, the first five largest stock holder account for 72% of all wheat stocks in 2017. By far, the largest fraction of total wheat stock is held by China with 47% followed by the U.S. with 10%, the EU and Russia with 5% and India with 4%. However, absolute values are not very informative with respect to the buffer function of storage. Thus, the ratio of stocks-to-consumption or stocks-to-use ratio (stu-ratio) is commonly used to express the vulnerability against production shortages. According to the USDA, China increased its stu-ratio over the last 10 years from 40% to more than 100%, while the EU, Russia and India remain at low levels below 20%.

2.2 Wheat trading on exchanges

Worldwide, wheat futures are traded on various exchanges, where different types of wheat are deliverable. Currently, the most important wheat exchanges are located in the U.S. and Europe. According to the CME Group, the marketplace is handling on average 3 billion contracts worth approximately $1 quadrillion annually. Other exchanges, which might become more important, are located in China and India. In general, exchange based trading tends to migrate to those trading venues where most liquidity is concentrated. Pooling liquidity in commodity futures presumes a highly standardised underlying in terms of quality, grading and delivery.[19] With highly standardised contracts, usually traders are exposed to a larger basis risk, since it is more likely that the trader's own wheat does not fully meet the required specifications for delivery at the exchange. Accepting a higher basis risk, however reduces the hedging effectiveness, resulting in a trade off between basis risk and liquidity, which has been noticed by Working (1953) very early.[20] He claimed that traders prefer a "poor hedge that is cheap to a more nearly perfect hedge this is relatively expensive" (p. 341). Contrary, Adjemian and Janzen (2017) argue that multiple futures contracts occur if traders prefer a lower basis risk while excepting higher liquidity costs.

Currently, the international and national (U.S.) benchmark for pricing wheat is the Chicago Mercantile Exchange (CME). Deliverable are the following classes of wheat: SRW, HRW, Dark Northern Spring and Northern Spring. Delivery locations include Chicago area, Indiana area,

[16]See WAOB (2018).
[17]See Vogel (1999).
[18]See WAOB (2018).
[19]See Zimmermann and Haase (2017).
[20]Morgan et al. (1999) provide a detailed discussion of these issues.

Toledo and Ohio area. Two other exchanges trade wheat in the U.S. but are less liquid. The Kansas City Board of Trade (KCBT) and the Minneapolis Grain Exchange (MGEX) differ from the CME in terms of wheat classes for delivery and respective delivery location. For example, the KCBT futures contract allows for delivery of HRW wheat in Missouri (Kansas City) and Kansas (Hutchinson, Salina/Abilene and Wichita), while the MGEX contracts allows for delivery of HRS wheat in Minneapolis, Duluth and Minnesota regions.

In Europe, there are mainly two important futures exchanges: the pan-European exchange (EURONEXT) located in Paris and the Intercontinental Exchange (ICE) located in London. However, both exchanges have a substantially lower degree of trading volume than their U.S. counterparts, see for example Haase and Huss (2017). Trading activity, in terms of open interest and volume, grows considerably faster in Paris compared to the U.S. markets[21], although on a lower level.

The European benchmark for the pricing of physical wheat is the EURONEXT's Milling Wheat No. 2 futures contract. This contract represents European soft wheat, most of which is harvested in July and requires a minimum protein content of 11.5%. The contract reflects the particular value for milling quality in Europe (France) and represents specific supply and demand factors, which may differ from those in the U.S. Delivery locations are at two silos in Rouen and one in Dunkirk on France's north coast.[22] UK Feed wheat futures are traded at the ICE and delivery takes place at a registered store on Great Britain's mainland.

Figure 2 shows the daily price development of September wheat contracts, traded in Chicago, Kansas, Minneapolis and Paris. In spite of quality differences mentioned above, the almost parallel price pattern clearly suggest that there is a common factor which drives all wheat prices.

In the following, the focus is set on the characteristics of the CME and the EURONEXT, while wheat contracts of other exchanges are analysed in order to provide complementary information.

2.3 Wheat futures price discovery and contract specifications

Adjemian and Janzen (2017) find that more than 80% of the wheat market price discovery is generated in Chicago. This finding is not surprising given the large trading activity in Chicago compared to other markets. Figure 3 shows the trading volume and open interest for the markets Chicago, Kansas, Minneapolis and Paris over the period 1995 to 2015. In spite of the large increase in the trading volume and in the open interest in Paris since 2006, the overall picture highlights the leading rule of the CME.

Although wheat prices follow a common trend, there are temporarily substantial price differences across various exchanges. Figure 4 shows the daily price spread development between the September Milling Wheat No. 2 contract, traded on the EURONEXT (CA), and U.S. September wheat contracts, traded on U.S. exchanges. A positive spread indicates a higher value of CA wheat (premium), while a negative spread the opposite. The figure shows, that price spreads are small on average (gray line), ranging between -25 and +50 USD per tonne. However, substantial divergences

[21] See Janzen and Adjemian (2017).

[22] Delivery at an approved silo in Rouen (France) and in addition at an approved silo in Dunkirk (France) from the September 2015 delivery month.

Figure 2: Wheat price development for September contracts at various exchanges

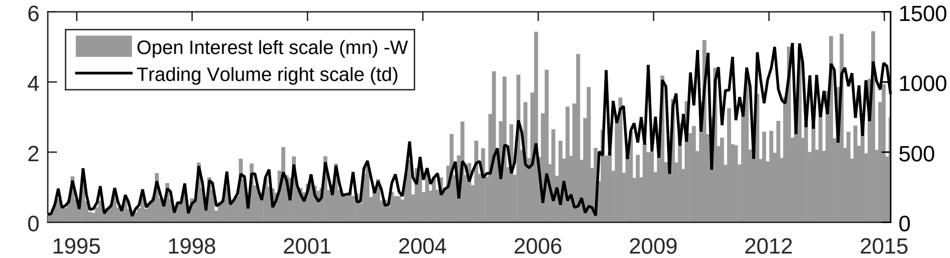
The figure shows the daily price development of September wheat contracts, denoted in USD per tonne. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. The contracts are held over one year form October to delivery in September. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT, MW denotes the wheat contract traded at the MGEX and CA denotes the wheat contract traded at the EURONEXT Paris.

up to ± 100 USD per tonne are observable between 2007 and 2008, and in Minneapolis -100 USD in 2017.

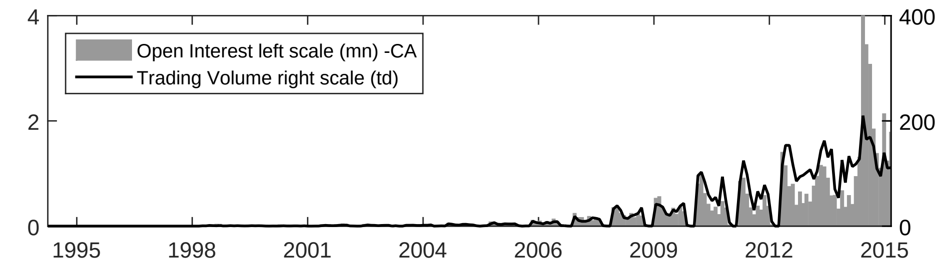
Such price differences between the exchanges might be best explained by the heterogeneity of traded wheat quality and to some extent by market frictions, like trade barriers or transport interruptions. As mentioned above, each exchange has established wheat contracts to value different wheat classes and their characteristics. In general, Chicago reflects the value of wheat with the lowest protein content (SRW, less than 11%), while Minneapolis values wheat with the highest protein content (HRS, min. 13.5%). Thus, price spreads reflect limitations of the substitutability between wheat classes, different quality specific supply and demand factors, and the limitation of spatial arbitrage.[23] Further, these specific market factors are time varying and determine the degree of volatility differences across wheat markets.

[23]for more details regarding spatial arbitrage in wheat markets it is referred to Brennan et al. (1997).

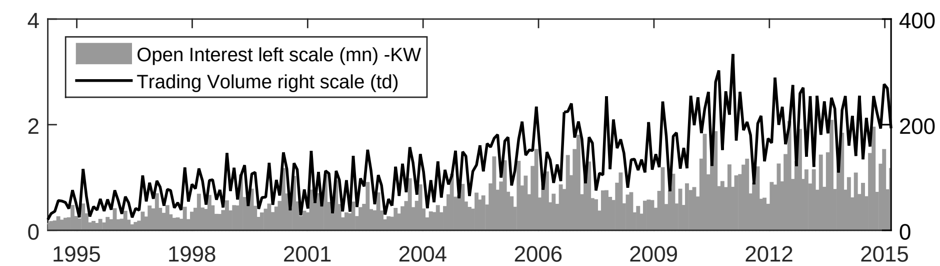
Figure 3: Trading Activity 1995 to 2015



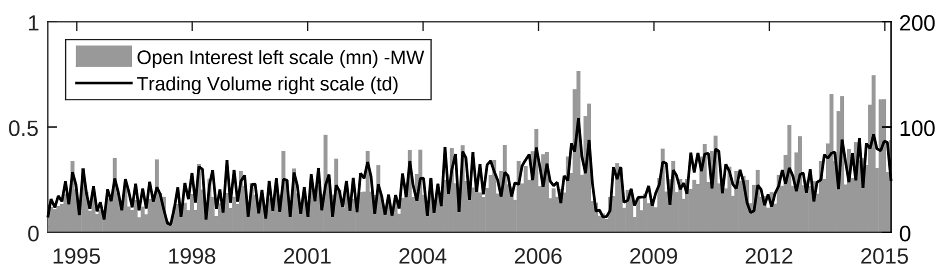
(a) Chicago Wheat (W), CME



(b) Paris Milling Wheat (CA), EURONEXT



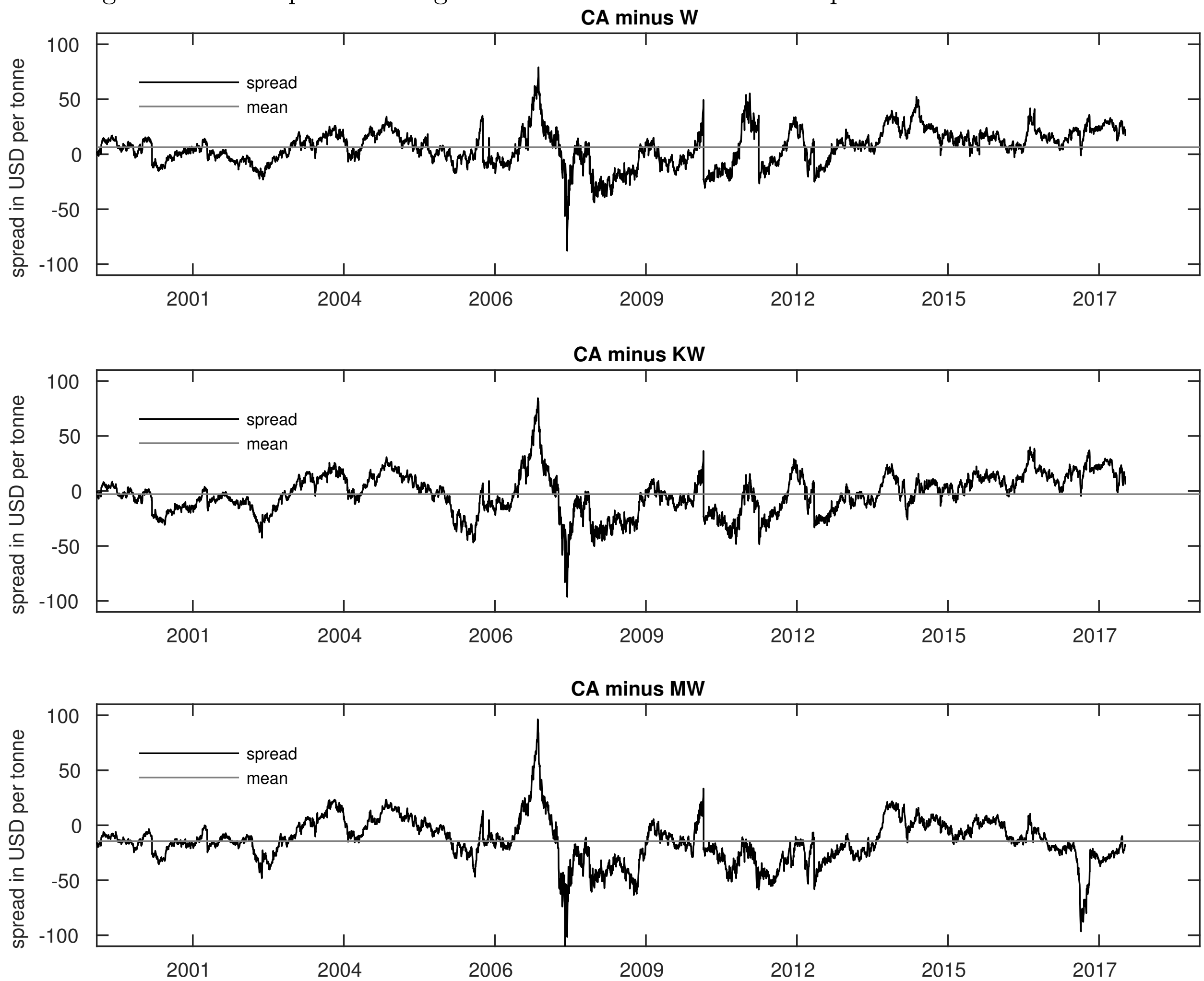
(c) Kansas Wheat (KW), KCBT



(d) Minneapolis Wheat (MW), MGEX

The figure shows monthly aggregate open interest in million contracts (left axis) and trading volume in thousands of contracts (right axis). The contract sizes for CA is adjusted to match the size of the contracts traded in the US.

Figure 4: Wheat price convergence between Paris and U.S. September wheat contracts



The figure shows the daily price spread development between the September Milling Wheat No. 2 contract traded at the EURONEXT (CA) and U.S. September wheat contracts. The price spread is denoted in USD per tonne. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. The contracts are held over one year form October to delivery in September. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX.

Table [5] in the Appendix provides a detailed overview of the contract specifications and market characteristics for wheat futures traded in Chicago, Paris, Kansas and Minneapolis. In brief, the most important differences of market and contract characteristics between Chicago and Paris are:

- lower protein content in Chicago compared to Paris,
- Chicago is quote in USD, Paris is quoted in EUR,
- settlement price is determined at different times: 1:20 p.m. Central Time (CT) Chicago versus 11:30 a.m. CT Paris,
- only partly overlapping trading periods,[24]

[24] A good overview of trading hours for U.S. wheat and Paris wheat futures is given by Janzen and Adjemian (2017), page 1198.

- larger contract size in Chicago with 136 versus 50 tonnes in Paris,
- CME delivery places are located in the U.S. vs. EURONEXT delivery places are located in France.

These specific characteristics determine the degree of substitutability between different wheat classes and help explain price and volatility differences.

2.4 Wheat option contracts

Wheat options analysed in this study are written on a pre-defined futures contract. Only this futures contract is deliverable when the option is exercised and not physical wheat, shipping documents or related delivery notes. Since the option's underlying itself is a tradeable contract, the option model simplifies to the standard option pricing model of Black (1976), who was the first to provide a formula to value commodity options in terms of the futures price.

Two aspects are important: first, the option's maturity is typically shorter than the maturity of the underlying futures contract. For example, all U.S. option contracts expire on the last Friday of the month prior to the futures expiration month, which is roughly half a month[25].

The time spread at the EURONEXT is slightly higher, i.e. the contract's expiry is normally the 15th day of the month preceding the delivery month of the futures contract, which is roughly 3 to 4 weeks.[26]

Second, all options are of American-style, i.e. the option may be exercised at any time up to its date of expiration.[27] For further details, refer to the respective exchange rule books of the CME, the KCBOT, the MGEX and the EURONEXT.

2.5 Summary market overview

Price and volatility differences between wheat futures traded at different exchanges might be best explained by contract and exchange specific characteristics, which determine the degree of demand substitutability between wheat varieties. Important differences of market and contract characteristics between Chicago and Paris are: i) the kind of wheat (quality and grading) allowed for delivery, ii) the contract's currency and iii) the degree of market integration.

Wheat option under this study are written on futures contracts and not on spot prices. As a consequence, the price characteristics of a futures contract must be considered to value an option on this futures, since spot price characteristics are already reflected in the futures price. Modelling the option value directly on a traded futures, simplifies substantially the option pricing model.

[25]See CME exchange rulebook, chapter 14A and MGEX Rulebook chapter 14, option specifications hard red spring wheat

[26]See EURONEXT, Technical specifications of the No. 2 Milling Wheat option contract, no. 10.2 Settlement procedure.

[27]See CME rulebook 14A01.H.Nature of Options on Wheat Futures, and EURONEXT Technical specicifactions of the no. 2 milling wheat option contract.

3 Historical wheat futures price volatility

This section focuses on the evolution of historical wheat **futures** price volatility traded on different commodity exchanges. Because, a futures contract is the option's underlying and therefore determines the option's value. Particular attention is paid to the comparison between Paris and U.S. September wheat futures. To analyse possible causes of variance difference between Paris and U.S. wheat, the focus is set on four impact factors, which are commonly cited as potential volatility drivers, namely:

- currency,
- quality and grading standards,
- liquidity and
- market integration.

In order to isolate the currency effect, Paris contracts, denominated in EUR, are additionally converted into USD. Further, to capture a broader range of grading standards, the sample is extended with wheat contracts traded on the KCBT (Kansas) and on the MGEX (Minneapolis). Liquidity effects are analysed by comparing two kinds of volatility estimators: the first is a standard measure based on returns, while the second estimator is based on price ranges as suggested by Parkinson (1980), i.e. using intra month high and low prices. Finally, market integration is analysed by searching for structural breaks in the variance differences.

3.1 Evolution of wheat price volatility

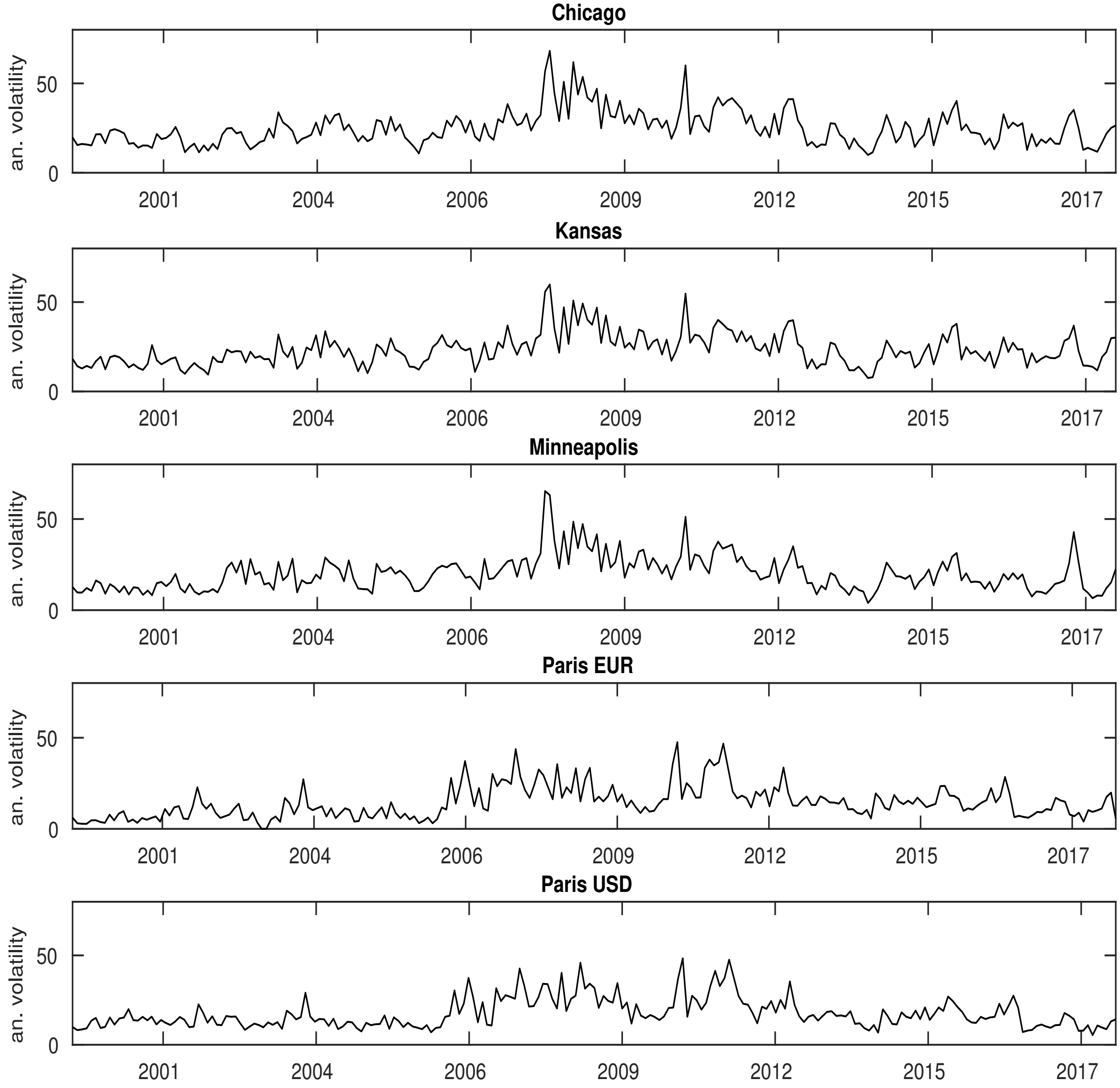
Figure 5 shows the monthly price volatility of September wheat futures contracts over the period October 1999 to March 2018. The first three panels represents U.S. wheat contracts denoted in USD, the last two panels represent the Paris wheat contract in EUR and USD, respectively. Visual inspection suggests that volatility of all markets varies significantly over time with a very similar pattern across all contracts. Additionally, on average smaller volatility is observed for Paris contracts denominated in EUR, at least until 2006.

3.2 Currency and quality/grading

In a next step, variance differences between U.S. and Paris September Wheat contracts are evaluated using a variance F-Test. Variance-ratios are built based on daily returns over the period October to September (contract maturity) of one marketing year. Table 1 contains the return based variance ratios for the observation years 1999 to 2017. The variance ratios between U.S. wheat in USD and Paris wheat in EUR are shown in the left part of 1, while the right part shows the ratio using Paris wheat in USD. A ratio larger than one indicates that the U.S. wheat has a larger variance during the observation year.[28]

[28] The volatility ratio is given by taking the square root of the variance ratio.

Figure 5: Monthly price volatility for September wheat contracts



The figure shows the evolution of monthly annualised volatility of U.S. wheat contracts, denoted in USD, and Milling Wheat No. 2 contracts traded on the EURONEXT (CA), denoted in EUR and converted to USD. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. Contracts are held over one year form October to delivery in September. At time of maturity, contracts are rolled-over into the next available September contract. The variance is measured over the period of one month using daily log returns and afterwards annualised by multiplying the daily volatility with $\sqrt{252}$. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX.

Three important results are found: first, variance ratios are in general lower when Paris wheat is converted from EUR into USD. In other words, the variance of Paris Wheat in USD is larger than in EUR. Taken the year 2000 as example, the variance (volatility) of Chicago wheat is 6.01 (2.45) times higher than the variance of Paris wheat in EUR, but only 1.69 (1.3) times higher by converting EUR contract into USD. This effect might be best explained by the observation that there is normally an inverse relationship between the dollar value and commodity prices. For example Headey and Fan (2008) or Akram (2009) found imperfect correlations, but a significant inverse relationship is often observed over time. Several reasons are commonly used to explain the inverse relation, but the main reason refers to the fact that international traded commodities are denominated in USD, i.e. the USD is the global numeraire for commodities. Once the USD devalues, an exporter requires a higher USD price in order to compensate for the exchange rate loss and vice versa.

Table 1: Variance F-test: September contract U.S. vs. Paris wheat return based

	Paris wheat in EUR			**Paris wheat in USD**		
season	**W**	**KW**	**MW**	**W**	**KW**	**MW**
1999/00	12.63***	8.83***	4.81***	2.79***	1.95***	1.06
2000/01	6.01***	4.76***	2.96***	1.69***	1.34***	0.84
2001/02	2.41***	2.26***	1.91***	1.47***	1.38***	1.16
2002/03	6.20***	5.53***	5.41***	3.28***	2.92***	2.86***
2003/04	4.25***	3.76***	2.76***	2.88***	2.55***	1.87***
2004/05	7.15***	5.38***	4.38***	3.93***	2.96***	2.41***
2005/06	3.89***	4.01***	3.09***	2.32***	2.39***	1.84***
2006/07	1.19*	0.95	0.74	1.12	0.90	0.70
2007/08	2.50***	2.00***	2.10***	2.10***	1.68***	1.76***
2008/09	3.01***	2.58***	2.16***	1.68***	1.44***	1.20*
2009/10	2.41***	2.06***	1.86***	1.85***	1.57***	1.42***
2010/11	1.33**	1.16	1.01	1.18*	1.03	0.90
2011/12	2.25***	2.15***	1.34***	1.79***	1.71***	1.07
2012/13	1.74***	1.55***	1.01	1.47***	1.31**	0.85
2013/14	2.71***	2.19***	1.65***	2.52***	2.03***	1.54***
2014/15	2.58***	2.26***	1.61***	1.90***	1.67***	1.19*
2015/16	1.90***	1.74***	0.92	1.65***	1.51***	0.80
2016/17	4.29***	4.57***	3.51***	3.62***	3.86***	2.97***
2017/18	2.50***	3.12***	1.11	3.44***	4.29***	1.52***

Season stands for the observed marketing year, which starts in October and ends in September of the following year ensuring that one marketing year of particular contract is covered. W refers to the wheat contract traded on the CME, KW is traded on the KCBT and MW is traded on the MGEX. Variances are calculated using daily log returns over the contract's marketing year. The table gives the one side F-value test statistics, i.e variance ratios between a September U.S. wheat contract and a September Paris wheat contract, implying a rejection of the null hypothesis of equal variances at the 10%/5%/1% level of significance (marked with $*/**/***$).

Second, Chicago show in general higher variance differences to Paris, while the difference is substantial lower by comparing Minneapolis with Paris and 8 out of 19 observation years have no significant differences[29], compare column 2 to 4 and 5 to 7. This result indicates the necessity of taking grading and quality standards into account.

Third, there is a tendency toward lower variance differences for all wheat classes after 2005, with an exception of 2016/2017. These results are consistent if May contracts are considered, see Table 6 in the Appendix. To sum up, the lowest variance difference is observed between Minneapolis and Paris wheat, both denominated in USD, see column 7 of Table [1], indicating that quality and currency are important factors to explain volatility differences.

3.3 Liquidity effects

In this study, market liquidity is proxied by terms of trading activity. There is a well documented positive relationship between trading volume and volatility in the academic literature.[30] The rationale is that more zero return days, i.e. non-trading days, resulting in downward bias of a return based volatility estimator.

Market liquidity in Paris is substantially lower compared to Chicago, see section 2.3. Thus, in addition to currency and quality effects documented in section 3.2 above, historically observed volatility in Paris is expected to be downward biased due to liquidity effects.

One possibility to estimate this bias is through using a range based volatility estimator as suggested by Parkinson (1980). In contrast to a return based estimator, which uses closing prices only, a range based volatility estimator relates on the entire trading range over an observation period. In this study intra-month high and low prices are used to calculate a monthly range based volatility estimator, which is more robust to non-trading days. Let P_τ be the futures price at time τ, then the price range over an interval $[t-1, t]$ is defined as

$$R_t = \max\left\{\ln(P_\tau)\right\} - \min\left\{\ln(P_\tau)\right\}, \text{where} \quad \tau \in [t-1, t], \tag{1}$$

is an unbiased estimator of volatility[31]. Most importantly, under the assumption that market participants concentrate their trades on days with sufficient liquidity, price dispersion is less biased than a return-based estimator. Further, in low-frequency contexts such as used in this study, the price range is a more efficient estimator of volatility than the price variance.[32] One drawback with range-based estimators in this context is related to its sensitivity to uncommon price spikes during the observation period, for example due to an unusual order, leads to an overestimation of the

[29] A rejection of the null hypothesis of equal variances at the 10%/5%/1% level of significance is marked with $*/**/***$.

[30] See for example Bessembinder and Seguin (1992) or Giot et al. (2010), who claims in addition that the trading volume-volatility relation does not hold for jumps.

[31] The relationship between a price range and the estimate of a price variance $\hat{\sigma}_P^2$, given the price, follows a simple diffusion model without drift: $\hat{\sigma}_P^2 = \frac{1}{4\ln 2}\left(\ln H_t - \ln L_t\right)^2$, where H_τ (L_τ) denotes the maximum (minimum) price over the period τ.

[32] For detailed explanations it is referred to Alizadeh et al. (2002) or Brandt and Jones (2006).

volatility.[33]

Overall, using range-based volatility estimators leads to similar results as those presented in Table [1], although observed variance differences are slightly smaller between Paris and Minneapolis. However, in some cases the observed variance in Paris is larger then in Minneapolis, compare [1] with Table [7] in the Appendix. This observation is explained by the occurrence of single prices spikes.

Given the large increase in Paris trading volume and the findings above, liquidity seems to be of minor importance to explain volatility differences, and therefore the focus of this study is set on return-based volatility estimators.

3.4 Market integration

The aim of this section is to analyse whether volatility difference between exchanges are also related to the level of market integration. Market integration refers to the degree of interconnection between markets and is an important aspect to describe differences in the market structure between markets. In a frictionless world, markets are perfectly integrated through arbitrage and prices of the same product move identical in all markets resulting in equal price-variances. However, markets are not frictionless and the amount and shape of market frictions ultimately determine the degree of market integration. In other words, if agents can not (or only partly) interact between markets, prices are determined independently from each other and markets are (partly) separated. Hence, price spreads, as a result of trade restrictions or barriers (taxes and tariffs), remain in the market, since they cannot be exploited by any arbitrage strategy.

Figure 4 in section 2.3 shows the wheat price convergence between Pairs and U.S. exchanges exists but varies over time. However, price convergence does not allow to draw any conclusion with respect to variance differences and their times series behaviour. In the following, it is analysed whether variance levels change over time and whether they are of transitory or permanent character. This distinction allows to identify possible market frictions.[34]

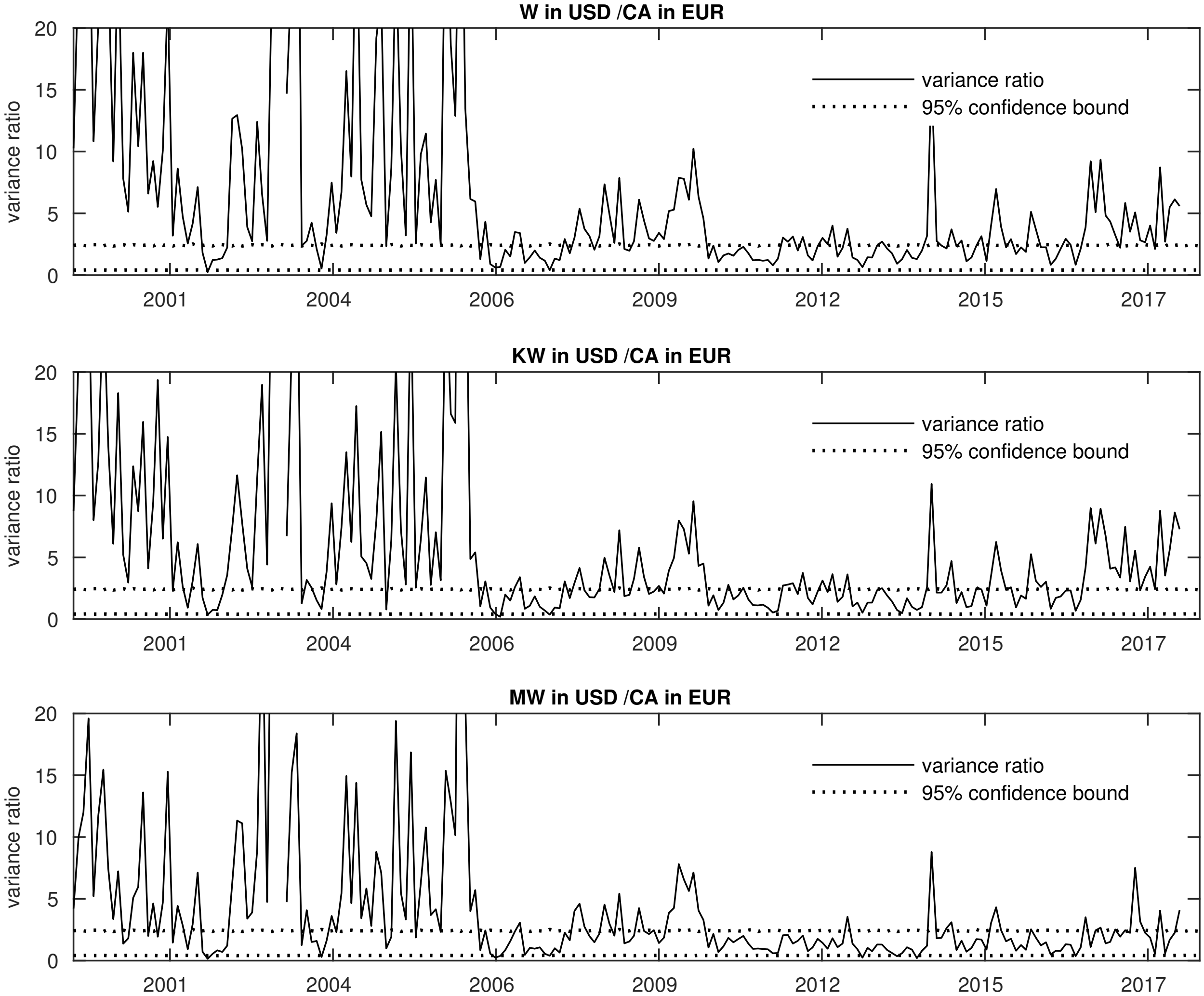
Figures 6 and 7 show monthly variance ratios between U.S. and Paris wheat contracts (black line) and the respective 95% confidence bound (dotted line). Variances are based on daily log returns of September contracts, which were held form October to their maturity in September. The observation period covers the years 1999 to 2017. In EUR denominated Paris wheat contracts are displayed in Figure 6 and Paris wheat converted in USD is shown in Figure 7.

A substantial break in all variance ratios is found around 2006. This break is also observable after accounting for currency effects (compare Figure 6 and 7). Focusing on variance ratios both contracts denominated in USD (Figure 7), before 2006 U.S. wheat contracts show mostly larger variances up to factor 15 (Chicago) or factor 10 (Minneapolis), implying a higher volatility factor of 3.9 for Chicago and 3.2 for Minneapolis. In contrast, after 2006 variance ratios decline substantially

[33]In other cases, such as limit orders and barrier options, such artificial prices might be of relevance to determine higher moments of the price distribution.

[34]For example, short term effects are caused by temporarily logistics disruptions, while import duties or trade barriers commonly trigger long term effects.

Figure 6: Return based variance F-test for September contracts between the U.S. and Paris (EUR)



The figure shows the evolution of monthly variance ratios between U.S. wheat contracts, denoted in USD and the Milling Wheat No. 2 contract traded at the EURONEXT (CA), denoted in EUR. All contracts expire in September. The contracts are held over one year from October to delivery in September. The return based variance is measured over the period of one month. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX.

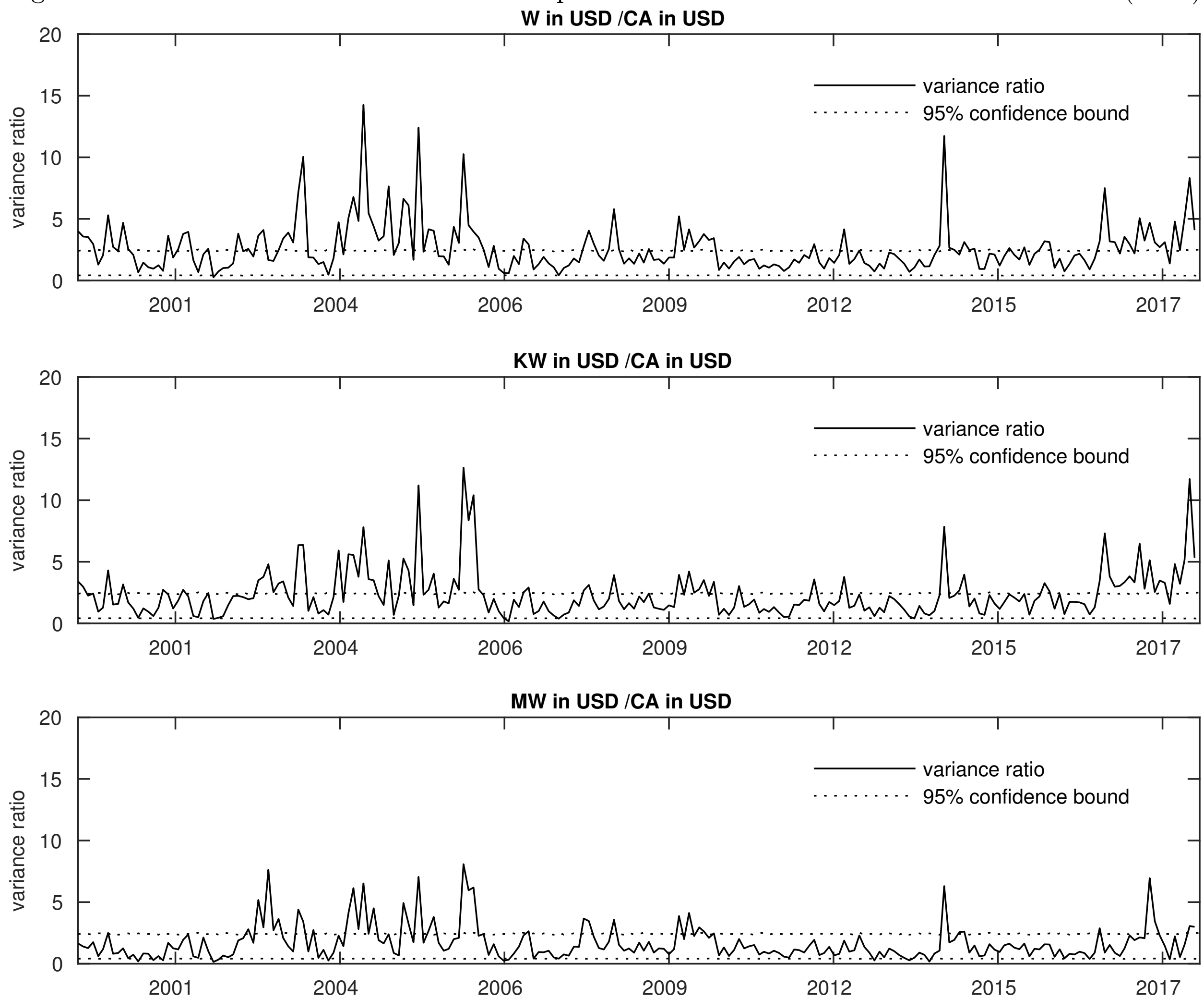
and are mostly insignificant, i.e. the null hypothesis of equal variances cannot be rejected. Another increase in the variance ratio is observable in 2017, while the duration of this break cannot be determined jet.

A possible explanation

A note at caution, a structural break does not allow for conclusions on causality. Rather, it helps to narrow down the period of time in which an event, that might had caused the break, had occurred. A very likely event in this respect is the reduction of EU import tariffs from 95 EUR per tonne to 12 EUR per tonne for low and medium quality of common wheat with a maximum import quota of 572.000 tonnes a year for the U.S.[35] In 2006, wheat prices where approximately 200 USD or 165 EUR, see Figure 2. An import tariff of 95 EUR corresponds to almost 60% of the U.S. wheat price,

[35] See EC (2006), page L 176/51

Figure 7: Return based variance F-test for September contracts between the U.S. and Paris (USD)



The figure shows the evolution of monthly variance ratios between U.S. wheat contracts and the Milling Wheat No. 2 contract traded at the EURONEXT (CA), converted into USD. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. The contracts are held over one year form October to delivery in September. The return based variance is measured over the period of one month. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX.

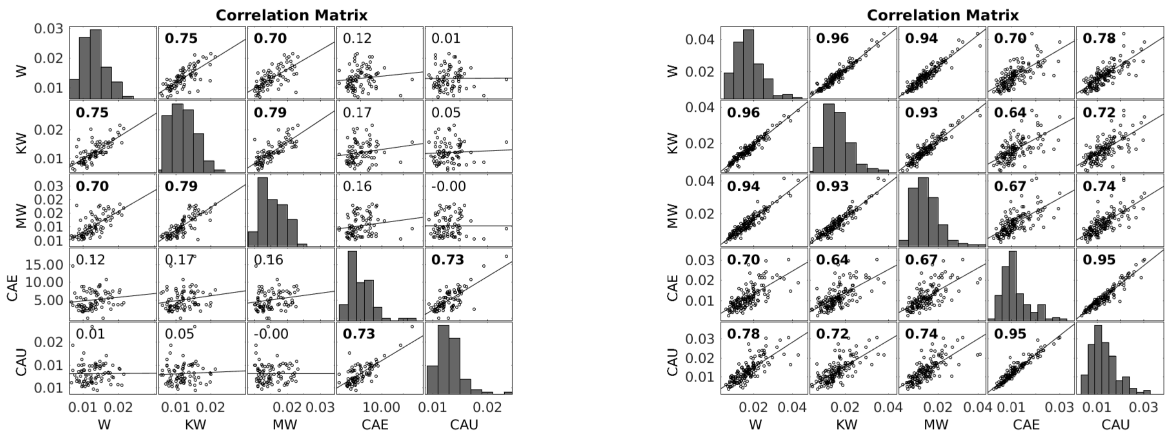
while 12 EUR corresponds to only 7%. Thus the high import tariff impair price arbitrage between U.S. and EU wheat until 2006. Further, the import tariff of 95 EUR per tonne becomes active by exceeding the applicable import quota during the year.[36]. Thus, the achievement of an import quota can be regarded as a trigger point for market integration. All results remain consistent using the alternative range-based volatility measure, see Figure 16 in the Appendix.

In a final step, the degree of market integration is analysed by comparing correlation coefficients between the variance of September wheat contracts of all four markets, taking into account the structural break in 2006. A graphical illustration, including respective distributions, is given in Figure 8. Subplot (a) contains the period between 1999 and 2006, while subplot (b) contains the

[36]See EC (2006)

period 2007 to 2017. As shown in subplot (a), variances are almost uncorrelated between Pairs and all U.S. contracts, while the Paris contract converted into USD shows a correlation of zero. This picture substantially changes after 2006, the correlation coefficients increase to 0.72 (KW) and 0.78 (W), which are all statistically significant. Further, the EUR denominate contract shows lower correlation coefficients to U.S. wheat contracts as the in USD converted contract.

Figure 8: Correlation between return based price variances for September wheat contracts



(a) Period 1999 to 2006

(b) Period 2007 to 2017

The figure shows the correlation coefficients of return based variances between the following September wheat contracts: Milling Wheat No. 2 contract traded at the EURONEXT denoted in EUR (CAE) and converted in USD (CAU), W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX. All contracts expire in September. Contracts are held over one year from October to delivery in September. The return based variance is measured over the period of one month. Coefficients which are significant different from zero are marked in bold.

3.5 Summary historical wheat price volatility

September wheat futures contracts traded in the U.S. and Paris show alternating periods of stronger and fewer volatility differences. This observation is analysed with respect to four potential impact factors, namely: currency, quality/grading standards, trading liquidity and market integration.

The exchange rate is one of the most important factor explaining volatility differences between U.S. and Paris wheat contracts.The volatility of a EUR quoted Paris wheat contracts increases when converting it into USD, since there is normally an inverse relationship between the dollar value and wheat prices quoted in USD.

Another important impact factor refers to quality standards, where lower variance differences are observed when the quality spread between contracts narrows. This is the case for Paris and Minneapolis wheat, where the lowest variance difference is detected.

Based on the findings of this study, liquidity plays a minor rule with respect to variance differences. Although, liquidity in Paris has been low in former times, an substantial increase in trading volume has been observed during the recent years, making the consideration of liquidity effects less important for current variance estimates. However, less liquidity, in terms of fast execution of large orders, might increase the costs of option replication, and thus would be normally reflected in a higher implied volatility.

Finally, variances between Paris and all U.S. wheat contracts have been substantially narrowed after 2006, i.e. with the reduction of EU import tariffs from 95 EUR per tonne to 12 EUR per tonne for low and medium quality of common wheat. Thus, the imposition or suspension of import quota, taxes or tariffs can be regarded as a trigger point for market integration. The degree of such market interventions determine whether and to which extent variances differences are affected.

4 Modelling seasonal pattern in historical wheat price volatility

There is extensive literature documenting seasonal behaviour of commodity prices. Fama and French (1987) and later Geman (2010) find evidence for seasonal pattern in spot prices for a large set of commodities. These findings are supported by studies focusing on agricultural commodities, e.g. Milonas (1991), Sørensen (2002), Geman and Nguyen (2005) or Paschke and Prokopczuk (2010).[37]

However, options under this study are written on futures contracts and not on a spot prices. The price of a futures contract represents the price of wheat delivered at a specific location at the time of the futures' expiration.[38] Any price expectations, such as seasonal price variability, are already reflected in the futures price and thus do not need to be taken into account to value the option.[39].

Many studies have documented that the volatility of a futures contract is time varying and has at least two deterministic components. The first refers to the well known "Samuelson effect" Samuelson (1965), which describes the empirical observation of increasing volatility as a futures contract approaches maturity. The second component refers to the observed seasonal pattern of a contract's price volatility over a year. Anderson and Danthine (1983) provide a theoretical foundation by relating (seasonal) volatility to (seasonal) information flow. For grain commodities, information flows tend to be highest during growing and harvest season of the crop and lowest during the dormancy period. Empirical evidence is provided among others by: Choi and Longstaff (1985), Fackler and Tian (1999), Sørensen (2002), Richter and Sørensen (2002), Back et al. (2013) or Arismendi et al. (2016).

In the following, the focus is set on the parametrisation of these two deterministic components, namely the time to maturity effect or "Samuelson effect" and seasonality.

[37]Energy markets show also strong evidence for seasonality in spot prices, see for example Borovkova and Geman (2006) or Tolmasky and Hindanov (2002).

[38]Wheat options traded on the CME and on the EURONEXT are described in section 2.4.

[39]More formally, a futures contract, as the option's underlying, is itself a tradeable and requires no capital. Thus, the futures price must have a zero drift under the risk neutral measure in order to avoid any arbitrage opportunities. It follows that all available informations are incorporated in the futures price, which itself determines the option's price.

4.1 Methodology

Seasonality and the time to maturity effect of volatility are analysed for wheat futures contracts traded on four commodity exchanges, namely CME, KCBT, MGEX, and EURONEXT. To obtain realised volatilities, daily log returns are calculated for September wheat contracts over the period 2006 to 2017. Futures are held for one year from October until delivery in September of the following year, i.e. one marketing year. Paris contracts are denominated in EUR. For comparable reasons EUR returns are converted into USD returns using Bloomberg exchange rates, where the time of price fixing is set as close as possible to the settlement time of Paris wheat futures.[40] The standard deviation is calculated for each calender month and annualised in order to facilitate:comparison with quoted implied volatilities and the interpretation of results[41].

A two step procedure is used to decompose the volatility time series into: i) the volatility level, i.e. the centre of gravity for a futures' volatility regarding the respective marketing year, and ii) a seasonal plus time to maturity component ($\varphi(t)$) describing the deterministic behaviour of the volatility around the futures' volatility level. Intuitively, the level volatility corresponds to the volatility, which would be observed without seasonality in the contract's price volatility. Following Geman and Nguyen (2005), the volatility $\sigma_{F,t}$ of a futures F at time t has two components:

$$\sigma_{F,t} = \bar{\sigma}_F e^{\varphi(t)}, \tag{2}$$

where $\bar{\sigma}_F$ is the futures' volatility level, and $e^{\varphi(t)}$ describes the seasonal behaviour around the futures' volatility level at time t. Taking logs on equation 2 yields the volatility decomposition:

$$ln(\sigma_{F,t}) = ln(\bar{\sigma}_F) + \varphi(t). \tag{3}$$

Further, two seasonal functions are specified:

$$\varphi(t) = \theta \sin(2\pi(t + \zeta)), \text{ and} \tag{4a}$$

$$\varphi(t) = c + \beta(T - t) + \theta \sin(2\pi(t + \zeta)), \tag{4b}$$

where $\varphi(t)$ denotes a sinusoidal function, θ denotes the amplitude parameter, i.e. the peak deviation of the function from zero, and ζ the respective phase parameter, i.e. where the oscillation is zero at time t. Equation 4a corresponds to the version of Geman and Nguyen (2005)[42] and is denoted with $Model - S$.

Equation 4b additionally contains the time to maturity effect, denoted with $Model - ST$, where

[40]Note, WM Reuters currency rates are the most representative rates, where the fixing takes place at 4 pm London time. However, settlement prices for Paris wheat contracts are determined with a time delay at 5:30 pm London time. In order to shrink this delay, the Bloomberg currency fixings (EUUS F133 curncy) are used since availability in June 2007 and before VW Reuters' rates are considered.

[41]The procedure to calculate log returns and monthly volatility is given in section 7 using equation 15 and 16, respectively.

[42]Note, Geman and Nguyen (2005) have additionally specified a mean reversion process in the futures' volatility. However, we skip the mean reversion component, since calibrated mean-reverting parameters are less reliable using a futures life time of only one year and 12 volatility observations per marketing year.

the constant c and $\beta(T-t)$ captures the impact of decreasing time to maturity, $(T-t)$ reflects the time to expiration in months.

Volatility level calibration

Note, the volatility level is not observable and therefore must be estimated from historical volatility observations. This can be done by smoothing the time series of empirical volatility observations, while ensuring that seasonal effects are eliminated.

The trend decomposition is done by smoothing the log volatility[43] time series using the robust version of 'Lowess', which assigns lower weight to outliers in the regression[44] The span is set according to the following rule of thumb, $span = 12/(N/2)$, which is roughly 0.16. Figure 9 shows the evolution of volatility levels ($\bar{\sigma}_F$) over the periods 2006 to 2017. In general, volatility of all contracts differ in their levels but show similar patterns. Volatility levels of Chicago (red line) and Kansas (red dotted line) contracts are similar, Paris in EUR (black dotted line) shows the lowest level. Interestingly, Minneapolis wheat (yellow line) and Paris wheat converted in USD (black line) are of similar volatility level, at least in 2010, supporting the assumption that these contracts are of comparable quality. Another possibility to filter out level volatility is by applying a simple moving average (MA) with a span of one season (12 months). However, visual inspection shows that the smoothing effect of a MA-procedure is less pronounced and, due to the procedure, the last 6 observations are lost. However, the impact on estimated seasonal parameters is rather marginal.

Deterministic component calibration

In a second step, seasonality and time to maturity parameters (Φ) have to be estimated. $Model-S$ requires the estimation of $\Phi \equiv \theta, \zeta$ and for $Modle-ST$ the parameters $\Phi \equiv c, \beta, \theta, \zeta$ have to be estimated to take additionally the time to maturity effect into account.

The set of parameters Φ are estimated by minimizing the root mean squared errors ($RMSE$) of following objective function:

$$
\begin{aligned}
\Phi_t^* &= \arg\min_{\Phi_t} \mathrm{RMSE}(\Phi_t) \\
&= \arg\min_{\Phi_t} \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left[ln(\sigma_{F,T,t}) - \big(ln(\bar{\sigma}_{F,T}) + \hat{\varphi}_t(\Phi_t)\big)\right]^2},
\end{aligned} \tag{5a}
$$

where, N refers to the number of observations and T stands for the contract's maturity. Further, imposing $\theta \geq 0$ and $\zeta \in [-0.5; 0.5]$ ensures the parameters' uniqueness.

4.2 Results on seasonal and time to maturity parameters

Panel A of Table 2 provides the results of the parameter estimation of $Model-S$ (equation 4a) and panel B those of $Model-ST$) (equation 4b). Starting with panel A, amplitude parameters (θ)

[43]Log volatility is used in order to assure that volatility must not be negative. This is ensured by using an exponential function in the option pricing formula for the deterministic component in the volatility process.

[44]The used method assigns zero weight to data outside six mean absolute deviations, see Cleveland (1981).

Figure 9: Return based volatility trend comparison

Monthly volatility trends are estimated using the robust version of Lowess. Trend estimations are based on log volatilities, which are subsequently re-transformed. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. Contracts are held over one year form October to delivery in September. The return based variance is measured over the period of one month. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX. CA-EUR denotes the Milling Wheat No. 2 contract traded at the EURONEXT and CA-USD denotes the Milling Wheat No. 2 converted into USD.

are all statistically significant at the 1% level ranging between 0.18 for Kansas and 0.24 for Paris in EUR.

Phase parameters (ζ) are significant and negative indicating that the oscillation starts in autumn. Including the time to maturity effect reduces the amplitude parameter but leaves the phase unaffected. These results are in line with the findings of Samuelson (1965), i.e. a negative β implies increasing volatility by decreasing time to expiration. Further, a larger β coefficient indicates a stronger reduction effect on θ (compare Chicago and Kansas), pointing out that seasonal effects are overestimated when the "Samuelson effect" is ignored.

Parameter interpretation

According to equation (2), parameters shown in Table 2 are based on log volatility components and thus can be interpreted as percentage deviations from the log volatility level. This facilitates the parameters' interpretation and simplifies their application for an option pricing model.

Figures 10 a-c illustrate effects of both $Model-S$ and $Model-St$ on the deterministic volatility using the estimated parameters of Chicago wheat shown in Table 2.

Table 2: Seasonality and time to maturity effects

	Panel A: $Model - S$		**Panel B:** $Model - ST$			
Exchange	θ	ζ	c	β	θ	ζ
Chicago	0.20***	-0.22***	0.15***	-0.27***	0.11***	-0.23***
Kansas	0.18***	-0.18***	0.04***	-0.09***	0.15***	-0.17***
Minneapolis	0.22***	-0.21***	0.08***	-0.14***	0.17***	-0.21***
Paris EUR	0.24***	-0.29***	0.19*	-0.25***	0.18***	-0.33***
Paris USD	0.20***	-0.28***	0.17***	-0.26***	0.13***	-0.32***

The table contains estimated sine parameters, which describe the oscillation (seasonal pattern) in the volatility of September wheat futures contracts, with the exception of Paris, where the November contract is used between 2008 and 2014. Panel A contains the estimation results of seasonal effects only, where the parameter θ represents the amplitude, i.e. the peak deviation of the function from zero, and ζ represents the phase, i.e. where the oscillation is zero at time t. Panel B contains in addition a constant parameter c and the time to maturity effect is captured by β. Parameters are estimated using de-trended log volatility over the period January 2006 to April 2018 and the corresponding 10%/5%/1% level of significance is marked with $*/**/***$, respectively.

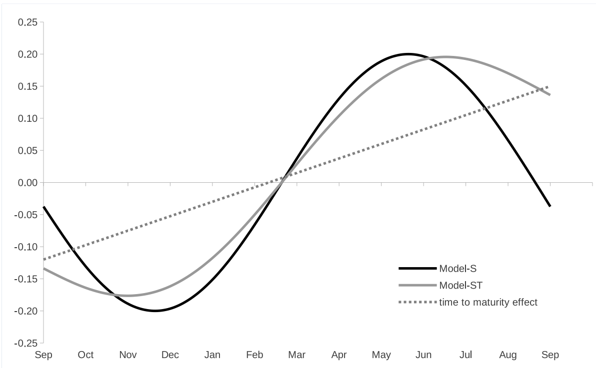
The development of deterministic log volatility components over the season September to August is shown in Figure (10 a). Consistent with other studies, both models show a similar pattern with the lowest volatility (ca. - 20%) around December and the highest (ca. + 20%) around harvest time in June and July. However, the impact differs by considering the time to maturity effect, which shows a stronger reduction in September and October and a less strong reduction in July and August. Figure (10 b) shows the development of (re-transformed) normal volatility using a level volatility of 20%. In contrast to log volatility, the normal volatility is more asymmetric, where the positive amplitude is more pronounced than the negative.

Although the Figures (10 a-b) show similar patterns and amplitudes, a strong difference occurs, once the level volatility is not observable. Suppose that a futures' volatility of 20% is estimated during the observation month September. In this case, $Model - S$ implies a lower volatility level of 20.5% compared to $Model - ST$ with 22.9%. This is because the seasonal volatility reduction in September of $Model - S$ is less pronounced than those of $Model - ST$. As a result, the volatility of $Model - ST$ is for all months during the marketing year larger than those of $Model - S$, where substantial differences occur during harvesting time with 27.5% and 23.6%, respectively. Hence, $Model - ST$ is the preferred model in this study.

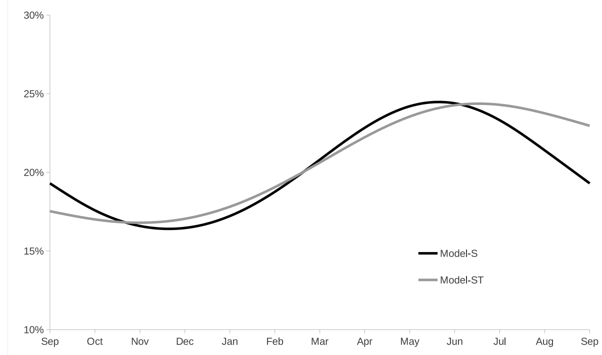
4.3 Summary

Empirical results, presented in this section, show that seasonality and time to maturity are important elements to understand and describe volatility of wheat futures prices during a marketing year. While the time to maturity effect is often ignored in commodity option pricing models, we show

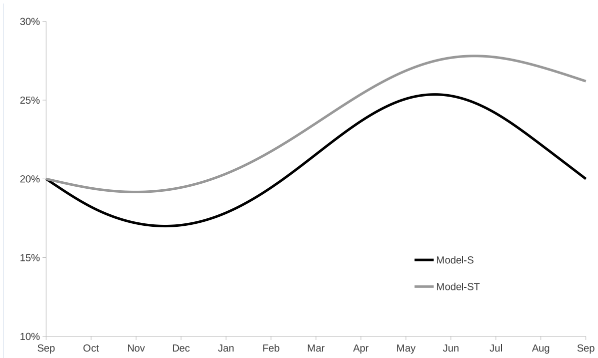
Figure 10: Comparing deterministic volatility components between $Model - S$ and $Model - ST$



(a) development deterministic log volatility component



(b) development seasonal volatility with $\bar{\sigma}_F(T) = 20\%$



(c) development seasonal volatility with $\sigma_F(t, T) = 20\%$

The figure shows the seasonal pattern of historical Chicago wheat futures price volatility with September expiration. The volatility development spans the season September to August, i.e. a marketing year. Seasonal parameters are taken from the Table 2 with a calibration window between 2006 and 2018.

that this effect should be explicitly considered[45] and is therefore the preferred model for capturing the main features of wheat futures' volatility in this study.

The introduced deterministically time varying volatility model helps to disentangle seasonal and time to maturity effects by wheat futures and provides a simple way to parametrize both effects. In contrast, analysing implied volatility is more complex due to the smoothing effect of time to integration[46]. Finally, the deterministic structure of the presented model facilitates the interpretation of wheat options' implied volatility.

5 The impact of USDA stocks-to-use ratios on wheat price volatility

This section provides a theoretical explanation on how inventory levels may affect the volatility of commodity prices. It also provides a theoretical foundation to describe economic states under which

[45]Schneider and Tavin (2018) have recently introduced a stochastic option pricing model for Crude Oil spread options, which accounts for the Samuelson volatility effect.

[46]For further details, it is referred to equation 11 in section 6

the marginal effect of supply/demand shocks on price fluctuations is particularly high.

5.1 Speculative storage models

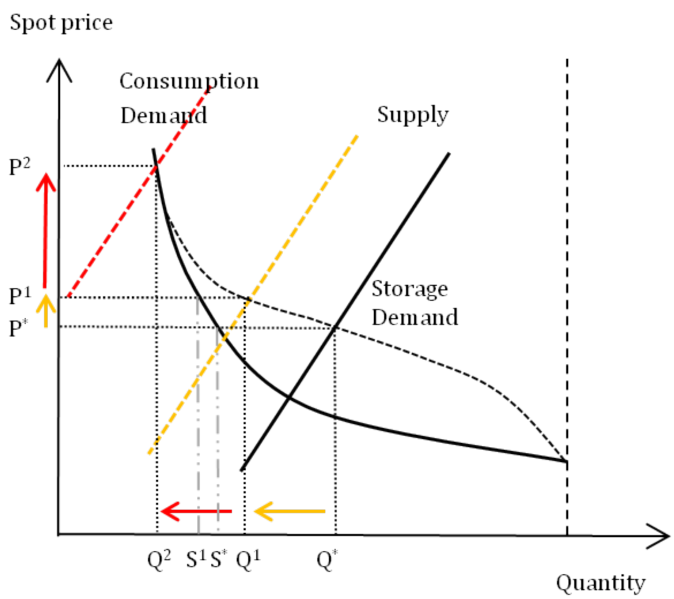
Storage models for intertemporal substitution play a crucial role in the context price fluctuations, because storage provides a buffer against supply and demand shocks. In particular, when inventories are low, a negative net supply shock cannot be compensated by depleting stocks, making the market more vulnerable to price spikes. These models explain the rule of stockholding and its impact on the price of a commodity in different market conditions.

Storage models were developed by Telser (1958), Brennan (1958),Williams and Wright (1991) or Deaton and Laroque (1996). Brennan's hypothesis is implicitly based on the assumption that the decision of storage operators - selling their inventories net forward - is based on speculative motives. The so-called "speculative" stockholders are actually intertemporal arbitrageurs and as such interact with producers and consumers alike. In particular, they create demand for a commodity in the current period and provide supply in a subsequent period, if the profit is expected to be positive. In doing so they smooth commodity prices. One important limitation in this context is that negative storage is not feasible for the market as a whole. Satisfying urgent consumption needs is only possible with commodities which have already been produced and stored, but it is not possible to borrow physical commodities from the future for current consumption. Thus, the intertemporal commodity transfer is unidirectional and price smoothing depends on the storage level.[47] When the current storage level is large enough and there are no capacity constraints, markets can respond to supply shocks by depleting or increasing inventories. Thus, the total net-demand is relatively elastic. However, when inventories are low, a negative net supply shock cannot be absorbed by depleting enough inventories leading to a larger price inelasticity.

Figure 11 illustrates the different impact situations. Consider an equilibrium price P* with a clearing quantity Q* and a sufficient amount of speculative storage, i.e. the amount between Q*and S*. Let us assume an inward shift of the supply curve from Q* and S1 (broken yellow line), where the reduction of the storage demand, (Q*-S*)-(Q1-S1), serves as a price buffer. As a result, the spot price increases only moderately from P* to P1 (yellow arrow). Now consider again an equal inward shift but now from Q^1 to Q^2 (broken red line), where only the amount of Q1- S1 can be taken out of storage. Hence, the combination of decreasing price elasticity and lower speculative stocks results in a considerable price increase from P1 to P2, see the red arrow. Spot price fluctuations are high during times of low stocks, where lower demand elasticity intensifies the impact. To sum up, competitive/speculative storage models predict for any storable commodity price spikes when inventories become low, since production cannot respond quickly to shocks and storage cannot be depleted to fill the supply gap. In a next step, it is empirically analysed at which storage level the volatility of wheat futures prices starts to increase as predicted by the model. Thereby, it will be distinguished between the volatility level and the seasonality.

[47]See Williams and Wright (1991) or Pirrong (2011).

Figure 11: The impact of different storage levels on the spot price



5.2 USDA wheat inventory data

The United States Department of Agriculture (USDA) is one of the most important information providers regarding U.S. and worldwide supply and demand estimates, crop conditions, planted acreage, stocks and other fundamental information, see Bunek et al. (2015) or Lehecka et al. (2014). These data are published in a monthly World Agricultural Supply and Demand Estimates (WASDE) report. In the following WASDE report monthly wheat inventory data are used to analyse the effect of inventory levels on the volatility of wheat futures.

WASDE report

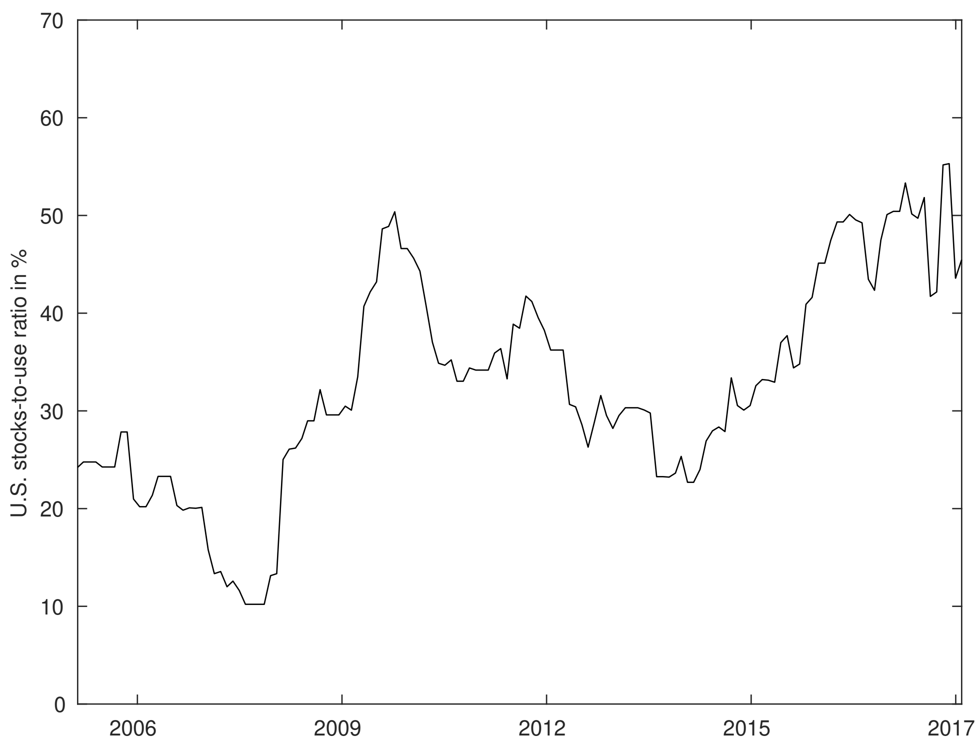
The monthly WASDE report includes the estimated ending stocks for wheat and other agriculture products based on a commodity-specific crop supply and demand forecast. The report gathers information from a number of statistical reports published by USDA and other government agencies, and provides a framework for additional USDA reports. Forecasts, derived during a specific month, are published by mid of next month. The expected carry-over projection, which is a point prediction to the end of the current marketing year, is updated each month until the end of April and switches afterwards into the projection of the following marketing year, where the old ending stocks become the new beginning stocks.[48] This accounting procedure is taken into consideration when monthly times series of inventory data are constructed. In particular, the marketing year has been changed in accordance with the analysed September wheat futures, i.e. ending projections are postponed until August. Ending stock predictions exclude seasonal components in the time series, which simplifies their application.

[48] Vogel (1999) provides a comprehensive explanation.

Inventory data show time trends as they grow in accordance with production and demand. To overcome this issue, inventory data are commonly scaled by consumption. The resulting ratio indicates the level of carry-over stock as a percentage of the total use and is known as the stocks-to-use ratio. For example, a 20% stocks-to-use ratio for wheat indicates that there are 75 days supply of wheat in reserve, see Chatnani (2010).

Another advantage of stocks-to-use ratio is the application as a well known indicator of vulnerability to price spikes.[49] According to Chatnani (2010), a stocks-to-use ratio for wheat under 20% has typically led to strong price advances. Bobenrieth et al. (2013) uses a more advanced model to calibrate critical values, but found similar levels slightly above 20%, while Greb and Prakash (2017) identified 37% for the U.S. and 18% on global level.

Figure 12: U.S. wheat all classes stocks-to-use ratio



WASDE U.S. wheat stocks-to-use ratios are taken form Blomberg covering the period January 2006 to December 2017. Monthly date are adjusted for the marketing year by postponing the ending stock projections from May to August.

In the following analysis, stocks-to-use data are taken form the category "U.S. Wheat Supply and Use", which are aggregates of all wheat classes produced and mainly used in the U.S. These data are downloaded from Bloomberg. Figure 12 shows the evolution of U.S. wheat stocks-to-use ratio covering the period January 2006 to December 2017. During the observation period, stocks-to-use ratios below 20% are recorded in only a few months.

[49] The theoretical foundation is given in section 5.1.

5.3 USDA stock to use ratio and volatility

In following, it is analysed at which critical stocks-to-use level the volatility of September wheat futures starts to increase and to which extent. In a first step only volatility levels are used[50] in order to disentangle level and seasonality effects. Used volatility levels are shown in Figure 9 on page 24. In a second step the focus is set on the relationship between stocks-to-use ratio and the volatility's seasonality, i.e. the parametrisation of the seasonal component.

However, the statistical analysis is constrained by the relatively low number of wheat shortages. Bobenrieth et al. (2013) found only 5 stock-outs for wheat in a 47 year sample and mentions the trade off between available data and statistical analysis:

> *"Large spikes are obviously quite rare in the available data. Even adding lesser spikes does not give us a sample useful for statistical analysis. Hence we must resort to a less formal analysis of the evidence."*, (Bobenrieth et al., 2013, p. 50)

Therefore, the relationship between stocks-to-use ratios and volatility's level is graphically examined by using box-plots. Figure 13 visualises the relationship for all wheat markets using 5 bins with a width of 10%, which corresponds to 36 days. Starting with the Chicago wheat subplot (a) of Figure 13, the highest volatility level is recorded for stocks-use-ratios below 20% (first bin) with a median annualised volatility level of more than 37%. Higher stocks-to-use ratios (bin 2 to 5) show substantial lower median volatility levels ranging between 20% and 25%. This picture is consistent for all other wheat markets, including EUR Paris (CA-EUR) for which generally a lower volatility level is observed, compare subplot (a) with (b) to (e).

These findings are in line with those of Bobenrieth et al. (2013) or Chatnani (2010) and underline the importance of stock-to-use ratios as an indicator for state dependent volatility. Using range based volatility estimators do not change these findings, implying model robustness against different volatility estimators, see Figure 17 in the Appendix.

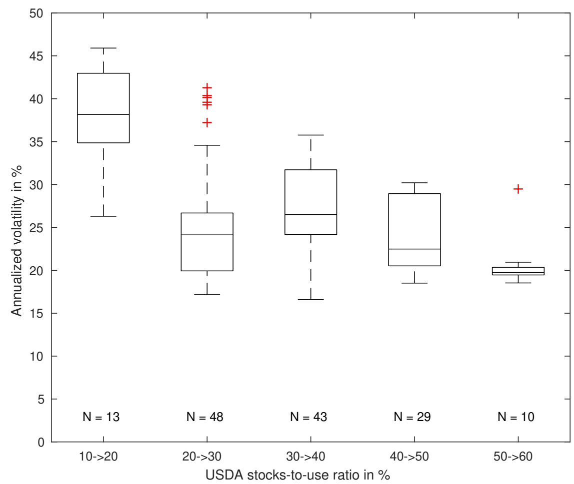
In a next step the relationship between stocks-to-use ratio and the volatility's seasonality is analysed by estimating state dependent seasonal parameters. This is done by splitting the data sample according to a critical stocks-to-use ratio. The respective critical value is now set to 25%, ensuring a sufficient number of observations in each subsample[51]. Using a critical value of 25% instead of 20%, increases the number of observation from 13 to 38 in the subsample, which represents the low stocks environment.

Estimation results are summarized in Table 3, where the low stocks environment is presented on the left panel of the table (columns 2 to 5) and the high stocks environment on the right panel (columns 6 to 9). Comparing the amplitude parameters (θ), parameters are significant in both states and approximately three times larger in the low stock environment, compare columns 2 with 6. Phase parameters (ζ) varies slightly between states without a systematic shift, compare across exchanges. More interestingly, the time to maturity effect is less pronounced and less significant in the low stock environment as in the high stock environment, compare columns 4, 5 with 8, 9.

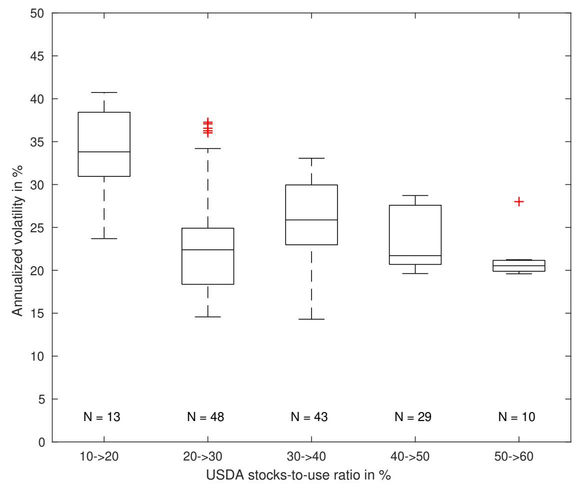
[50] The decomposition is done according to equation 3 using the robust version of "Lowess", see page 22.

[51] Bobenrieth et al. (2013) found critical values near 22%.

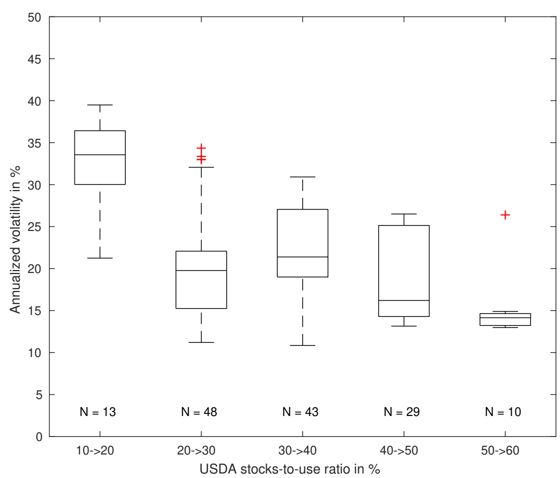
Figure 13: Impact stocks-to-use ratio on return based volatility



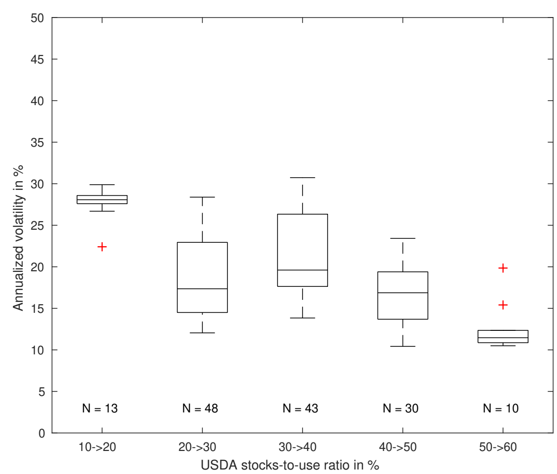
(a) Chicago Wheat (W), CME



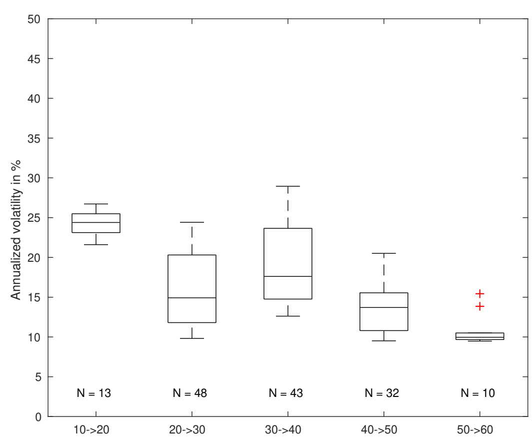
(b) Kansas Wheat (KW), KCBT



(c) Minneapolis Wheat (MW), MGEX



(d) Paris Milling Wheat (CA-USD), EURONEXT



(e) Paris Milling Wheat (CA-EUR), EURONEXT

WASDE U.S. wheat stocks-to-use ratios are taken form Blomberg covering the period January 2006 to December 2017 and are adjusted for the marketing year by postponing the ending stock projections from May to August. Monthly volatility levels are estimated using the robust version of Lowess as described in section 4, see figure 9.

To sum up, seasonality is dominated by the sine function when stocks are low, while the time to maturity effect is under represented. In contrast, when stocks are adequate, the time to maturity effect becomes more important, while the seasonality effect decreases.

Table 3: Seasonality and trend parameters of wheat futures volatility

	Stocks to use ratio $< 25\%$				**Stocks to use ratio $\geq 25\%$**			
	θ	ζ	β	c	θ	ζ	β	c
Chicago	0.21***	-0.22***	-0.12*	0.05	0.10	-0.23***	-0.31**	0.17*
Kansas	0.31***	-0.17***	0.00	-0.06*	0.12***	-0.17***	-0.13***	0.07***
Minneapolis	0.36***	-0.18***	0.05	-0.13**	0.13***	-0.24***	-0.22***	0.13***
Paris EUR	0.26***	-0.38***	0.03*	0.07***	0.15***	-0.28***	-0.37***	0.24***
Paris USD	0.28***	-0.34***	0.17***	-0.05	0.09***	-0.26***	-0.42***	0.26***

The table contains estimated sine and time to maturity parameters, which describe the oscillation (seasonal pattern) in the volatility of September wheat futures contracts. The parameter θ represents the amplitude, i.e. the peak deviation of the function from zero, ζ represents the phase, i.e. where the oscillation is zero at time t, the constant parameter is denoted with c and the time to maturity effect is captured by β. Parameters are estimated using de-trended log volatility over the period January 2006 to April 2018 and the corresponding 10%/5%/1% level of significance is marked with $*/**/***$, respectively.

Parameter interpretation

Figure 14 illustrates the effect of state dependent seasonality using the estimated parameters of Chicago wheat, presented in Table 3, with a level volatility of 37% in the low stock environment and 25% otherwise, corresponding to findings shown in Figure 13.

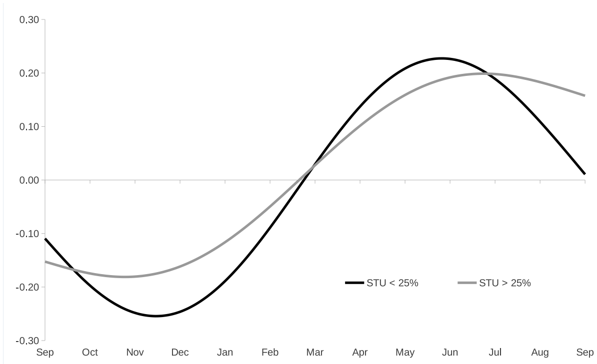
Focusing on subplot (a), the seasonality in a low stock environment shows a more pronounced amplitude (black line) with ± 22% between summer and winter compared to + 20% and − 15% in a high stock environment (grey line). Further, the volatility peak in a low stock environment is around harvest, while when stocks are high the volatility peak occurs around contract's maturity.

Adding state dependent volatility levels (subplot (b)) provides an interesting picture, which shows substantial volatility spikes around harvest. In a low stock environment volatility ranges between 32% in December and increases up to 50% in June/July (black line), while in a high sock environment volatility ranges only moderate between 25% and 35% (grey line).

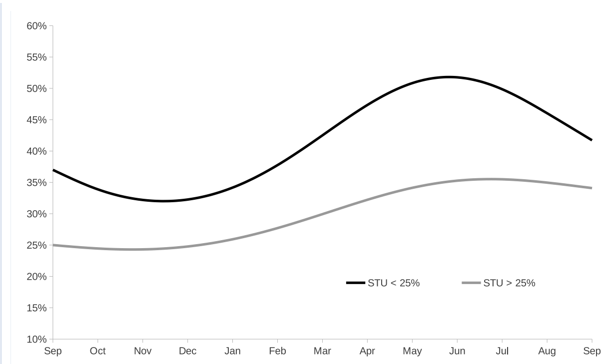
5.4 Summary

The introduced deterministic model provides large volatility spikes up to 50% around harvest time when the stocks-to-use ratio is below 25%. The advantage of the deterministic approach in this context is quiet obvious. Because, using stocks-to-use ratios, as a fundamental driver of volatility and seasonality, provides a profound basis for modelling extreme wheat price volatility by analysing current supply, demand and inventory conditions. The seasonal parameters can be chosen a priori according to how the user expectations of fundamentals. A major advantage of the introduced deterministic modelling approach in this study is its ease in its day-to-day practical application. Additionally, the low number of parameters makes the model less vulnerable to over-parametrization.

Figure 14: State dependent seasonal volatility component



(a) seasonal log volatility



(b) seasonal volatility with $\bar{\sigma}_{F(T)} = 25\%$, if $STU \geq 25\%$; and $\bar{\sigma}_F(T) = 37\%$, if $STU < 25\%$

The Figure shows the seasonal pattern of historical Chicago wheat futures price volatility with September expiration. The volatility development spans the season September to August, i.e. a marketing year. Seasonal parameters are taken from the Table 3, where the sample is split according to a critical stocks-to-use ratio (STU) of 25%. The annualised volatility level is set to 37% when $STU < 25\%$ and 25% otherwise. The calibration window spans the period between 2006 and 2018.

6 Option pricing models, implications and extensions

6.1 Literature overview

There is extensive literature covering seasonality in commodity prices and price volatility. Seasonality in the price volatility of a specific futures contract was mentioned early by Choi and Longstaff (1985). The main argument is that supply for most agricultural goods is determined by harvesting cycles leading to higher volatility during the growing and harvest season. For example Fackler and Tian (1999) analysed the characteristics of soybean futures and options based on a spot price process. He found price mean reversion to a seasonally varying mean as well as seasonal variation in the volatility. Sørensen (2002) adds a deterministic seasonal component to the constant volatility two-factor model of Schwartz and Smith (2000) and estimates the model parameters for corn, soybean and wheat futures.

Richter and Sørensen (2005) estimate a spot price model for soybean acknowledging that commodities exhibit seasonal effects in the spot price level and volatility. Koekebakker and Lien (2004) fit market option prices on wheat futures. They include seasonal and maturity effects of the volatility specification to the jump-diffusion option pricing model from Bates (1991). Geman and Nguyen (2005) focus on the soybean spot price process and include a deterministic component to describe time varying volatility. Back et al. (2013) extend a one- and a two-factor spot price model by allowing for seasonal changes in the volatility during a calendar year and apply their model to corn, soybean, heating oil and natural gas options. Arismendi et al. (2016) propose a seasonally varying long-run mean variance process and apply their seasonal stochastic volatility model to natural gas and corn futures options.

The main commonality of these models is that options are valued on spot prices with seasonal characteristics. However, seasonality in the spot price process is already reflected in the futures price, and therefore does not need to be taken into account, see Haug (2007) p. 411. Another disadvantage refers to the time to maturity effect of futures, which is not reflected in the spot price process and therefore ignored in many applications.

When modelling an option directly on a futures price, one reduces substantially the number of model parameters and therewith the risk of over parametrization. For example the well known model of Miltersen and Schwartz (1998) needs in total 8 parameters (3 volatilities, 3 correlations and 2 speed of mean reversion parameters), which has to be calibrated on a reliable database for calculating an European option price on commodity futures. Back et al. (2013) using up to 6 parameters, which are calibrated on the cross section of implied volatilities, i.e. options on futures contracts with different expirations. However, as shown by Fackler and Tian (1999), each contracts maturity has their own seasonal pattern, and thus calibrating parameters across maturities leads to distorted parameter values.

6.2 The model

The focus is set on the development of an option pricing model which reflects the main characteristics of wheat futures as described in section 4. Special emphasis is placed on the unique characteristics of (i) the market for such options, and (ii) the wheat market in general, both of which have an important influence on the modelling approach of the pricing model. The study intentionally focuses on a pricing approach that can easily be applied in daily business. Additionally, the following main aspects are considered:

- modelling an option directly on a futures price process,
- incorporating seasonality and the time to maturity effect,
- the seasonal amplitude depends on the volatility level.

As basis model to be extended is the well known Black (1976) option price model on futures contracts. In a risk neutral world the drift of the futures equals zero. Consequently, Black (1976) assumes that under the risk-neutral measure the futures price dynamics has the following form:

$$dF_t = F_t \sigma dZ_t^Q, \tag{6}$$

where dZ^Q is a standard Wiener process under the risk neutral measure representing futures return innovations and the σ is the constant volatility of the futures' returns. Under the risk neutral measure, F is to be a Q-martingale and the drift of the price process F under Q must be zero[52] Black's option pricing formula for call (C) and put (P) options on futures, with initial price F and

[52]More details with respect to the martingale representation theorem are given by Duffie (2003) on page 673 ff.

a strike price K, is given as:

$$\begin{aligned} C &= e^{-rT}\left[FN(d_1) - KN(d_2)\right], & (7)\\ P &= e^{-rT}\left[KN(-d_2) - FN(-d_1)\right], & (8)\end{aligned}$$

with

$$\begin{aligned} d_1 &= \frac{ln(\frac{F}{K}) + (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}},\\ d_2 &= \frac{ln(\frac{F}{K}) - (\frac{\sigma^2}{2})T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}\end{aligned}$$

where r denotes the risk free interest rate, σ denotes the constant volatility of the relative price change of the underlying futures contract, and $N(\cdot)$ stands for the cumulative normal distribution function.

While Black (1976) uses a constant volatility, we follow the idea of Geman and Nguyen (2005) or Back et al. (2013), and replace the constant volatility σ with the deterministic time varying volatility σ_s as described by equation 2. Hence, the price dynamic under the risk neutral measure is given by:

$$\begin{aligned} dF_t &= F_t\sigma_{s,t}dZ_t^Q,\\ &= F_t\bar{\sigma}e^{\varphi(t)}dZ_t^Q, & (9)\end{aligned}$$

with

$$\varphi(t) = \theta\sin(2\pi(t+\zeta)), \tag{10}$$

where $\bar{\sigma}$ denotes the volatility level, which is assumed to be constant over the options life time. The sinusoidal function, which describes the deterministic behaviour around the futures' volatility level, is represented by $\varphi(t)$, where θ denotes the amplitude parameter, i.e. the peak deviation of the function from zero, and ζ the respective phase parameter, i.e. where the oscillation is zero at time t. The following model extension accounts for the time to maturity effect:

$$\varphi(t) = c + \beta(T-t) + \theta\sin(2\pi(t+\zeta)), \tag{11}$$

where the constant c and β captures the impact of decreasing time to maturity and $T-t$ reflects the time to expiration. The seasonal and time to maturity parameters can be calibrated to historical data,as for example done in section 4 and 5, or to subjective beliefs of a trader.

The European call option price C is calculated using the standard Black's formula by accounting for a deterministic time varying volatility σ_s according to:

$$\sigma_s = \sqrt{\frac{1}{T-t}\int_t^T \bar{\sigma}^2 e^{2\varphi(\tau)}d\tau} = \bar{\sigma}\sqrt{\frac{1}{T-t}\int_t^T e^{2\varphi(\tau)}d\tau}, \tag{12}$$

where $\bar{\sigma}$ denotes the volatility level. The integral can be solved numerically, e.g. with Romberg's method or the trapezoid rule. Finally, the price for a call C and a put P option is then given by:

$$
\begin{aligned}
C &= e^{-r(T-t)}\left[FN(d_1) - KN(d_2)\right], && (13)\\
P &= e^{-r(T-t)}\left[KN(-d_2) - FN(-d_1)\right], && (14)
\end{aligned}
$$

with

$$
\begin{aligned}
d_1 &= \frac{ln(\frac{F}{K}) + \frac{\bar{\sigma}^2}{2}\int_t^T e^{2\varphi(\tau)}d\tau}{\bar{\sigma}\sqrt{\int_t^T e^{2\varphi(\tau)}d\tau}},\\
d_2 &= \frac{ln(\frac{F}{K}) - \frac{\bar{\sigma}^2}{2}\int_t^T e^{2\varphi(\tau)}d\tau}{\bar{\sigma}\sqrt{\int_t^T e^{2\varphi(\tau)}d\tau}}\\
&= d_1 - \bar{\sigma}\sqrt{\int_t^T e^{2\varphi(\tau)}d\tau}.
\end{aligned}
$$

The presented closed form solutions only exist for European options, while options under study are of American-style. An American option can be exercised at any time until expiration, which adds freedom but complicates the valuation. Conceptually, the valuation of American options would add to the precision in a practical application.[53] Doing so substantially increases the numerical complexity of the problem at hand and is beyond the scope of this study. Additionally, it is not guaranteed that the algorithm converges to the true option value in either case.[54] Furthermore, only at the money options with a time to maturity of less than one year are considered for which the American-style approach is of limited importance with respect to accurate option price. Given the estimation risk for the American-style approach and the limited price impact for options under this study, a robust estimation of the option price (i.e., via European Options) is preferred.

7 Empirical model validation

Crop insurance contracts under this study are typically valued in autumn, i.e. before sowing, and expire after or around harvesting time. It is assumed that the respective risk is kept over the contract's lifetime, which is roughly one year. The focus is therefore set on the comparison of realised volatility with predicted volatility over the contract's life time, i.e. from valuing until the contract's expiration.

It should be mentioned that this procedure differs from those typically used in academic literature, where anticipated expected volatility is at the centre of interest and not the realised volatility. Because, testing the accuracy of an option pricing model is related to the observed (traded) option price and therewith to its implied volatility, see for example Christoffersen and Jacobs (2004) or Arismendi et al. (2016).

[53] An analytic approximation of American option values is provided by Barone-Adesi and Whaley (1987).
[54] For further details, it is referred to Haug (2007), page 97 ff.

Ultimately, this section focuses on forecast properties of the introduced deterministic volatility model by comparing realised volatility with model predictions and market anticipated expected volatility, i.e. implied volatilities taken from traded options.

7.1 Methodology and data description

The following three volatility forecast models are used: i) a 30-day rolling historical volatility (σ_{30d}); ii) volatility based on the $Model - ST$, i.e. seasonality model including the time to maturity, (σ_{ST}); and iii) a volatility based on the conditioned version of the $Model - ST$ using 25% as critical value of the stocks-to-use ratio ($\sigma_{ST_{stu}}$). Of particular interest is the period of fall, when expected wheat price volatility is used as input parameter to determine the price of a crop insurance contract. Thus, only the months September, October and November are considered to derive volatility forecasts.

Implied volatilities (σ_{imp}) are based on at-the-money September wheat options. The option's underlying is a wheat futures contract, which expires in September of following year. Respective daily implied volatility data with 100% moneyness are taken form Bloomberg, where the reference model of Black (1976) is used to calculate the implied volatility [55].

In a first step, daily log returns ($r_{F_{T,t}}$) of September wheat futures are calculated according to:

$$r_{F_{T,t}} = ln\left(\frac{F_{T,t}}{F_{T,t-1}}\right), \tag{15}$$

where $F_{T,t}$ is the futures price at time t with a expiration date T. Contracts are held from beginning September to end of November, i.e. the time where crop insurance contracts are priced. The observation sample spans the period from 2006/07 to 2016/17 and includes 11 marketing years.

In a second step, log returns over a period of 30 days, starting 30 business days prior to the observation date, are used to estimate rolling historical volatilities using the following formula:

$$\sigma_{30d} = \sqrt{252}\sqrt{\frac{1}{N}\sum_{t}^{N}\left(r_{F_{T,t}} - \bar{r}_{F_T}\right)^2}, \tag{16}$$

where N is the number of business days in the period under study, here 30 days, and $\bar{r}_{F_T}$ is the average log return over N days, i.e. form $t-1$ to $t-N$, and the term $\sqrt{252}$ annualises the volatility estimator[56]. This procedure provides on average 35 daily volatility forecasts per season, which are further assumed to be constant over the option's life time.

Further, the daily price volatility ($\sigma_{30d,t}$) is used to calculate daily volatility forecasts based on $model - ST$ and its conditioned version using the φ-parameters presented in Table 2 Panel B and Table 3, respectively. In accordance with the findings in section 5 the critical level of the stocks-to-use ratio is set to 25%.

The following procedure is applied: first, calculating the volatility level $\bar{\sigma}$ using equation 3 and

[55] See Bloomberg documentation page 24. The used download shortcut is: $HIST\_CALL\_IMP\_VOL$.

[56] 252 are number of trading days per year.

setting σ_{30d} equal to $\sigma_{_{F,t}}$, according to:

$$ln(\bar{\sigma}) = ln(\sigma_{30d,t}) - \varphi(t). \tag{17}$$

Second, two functions describing the volatility's seasonal and time to maturity behaviour are specified, where the first $\varphi(\tau)$ refers to $Model - ST$ and the second $\varphi(\tau_{stu})$ refers to $Model - ST_{stu}$ as the conditioned version. Hence, the volatility forecast is performed using the following equation:

$$\sigma_{ST} = \bar{\sigma}\sqrt{\frac{1}{T-t}\int_t^T e^{2\varphi(\tau)}d\tau}. \tag{18}$$

Replacing $\varphi(\tau)$ with $\varphi(\tau_{stu})$ provides the conditioned volatility forecast $\sigma_{ST_{stu}}$ with respect to the stocks-to-use ratio.

Finally, realised volatility σ_{real} is calculated on a daily basis according to:

$$\sigma_{real} = \sqrt{252}\sqrt{\frac{1}{N}\sum_t^N \left(r_{_{F_{T,t}}} - \bar{r}_{_{F_T}}\right)^2}, \tag{19}$$

where N is the number of business days until the option's expiration T and $\bar{r}_{F_T}$ is the average log return over the remaining option's life time, i.e. form t to T. Again, the term $\sqrt{252}$ annualises the volatility. This procedure ensures that the same time horizon of realised, implied and predicted volatility is used.

7.2 Results

Left hand side of Table 4 displays averages of implied, predicted and realised annualised volatilities of all exchanges under study. The main results can be summarized as follows: first, implied volatilities $\sigma_{_{imp}}$ (column 2) of Chicago and Kansas underestimate realised volatilities $\sigma_{_{real}}$ (column 6), while an overestimation is observed for Minneapolis and Paris. Second, historical 30 days rolling price volatilities $\sigma_{_{30d}}$ (column 3) underestimate realised volatilities for all markets, where the largest gap is found for Chicago wheat with 6% points or more than 20%. Third, predicted mean volatility of the $Model - ST$ and its conditioned version (column 5 and 6, respectively) are close to the mean of realised volatility with a small negative bias. Both forecasts are almost identical, which might be best explained by the low number of realised stock-outs during the sample period.

Overall, $Model - ST$ has marginal superior predictive power in comparison to implied volatility, in particular for Paris, compare column 6 with columns 2, 4 and 5.

However, superior mean forecasts may come at the cost of larger forecast errors. In a next step, root mean squared errors (RSME) are analysed, which is a widely used procedure to compare the fit of models. RSMEs are calculated according to:

$$RSME_t = \sqrt{\frac{1}{N}\sum_{t=1}^{N}\left[\sigma_{F,t-T} - \hat{\sigma}_{F,t-T}\right]^2}, \tag{20a}$$

where, $\sigma_{F,t-T}$ denotes the realised volatility between the observation date and the option's expiration and $\hat{\sigma}_{F,t-T}$ denotes the model forecast, N refers to the number of observations and T stands for the options maturity, while $t-T$ denotes the time to option's expiration, i.e. the prediction horizon.

Calculated RSME results are presented on the right hand side of Table 4. Overall, implied volatilities provide the smallest RMSEs ranging between 0.08 for Minneapolis and 0.05 for all other markets, see column 7. Slightly larger RMSEs are found for the unconditioned ($Model-ST$) and conditioned ($Model-ST_{stu}$) volatility forecasts. RMSEs ranging between 0.06 for Chicago and 0.09 for Kansas. The largest RMSEs are found for the rolling historical 30 days average price volatility σ_{30d}, compare columns 7 to 10.

Table 4: Comparison of realised volatility with model forecasts and implied volatility

	mean					RMSE			
Exchange	σ_{imp}	σ_{30d}	σ_{ST}	$\sigma_{ST_{stu}}$	σ_{real}	imp	$30d$	ST	ST_{stu}
Chicago	0.27	0.23	0.28	0.28	0.29	0.05	0.08	0.06	0.06
Kansas	0.26	0.24	0.29	0.29	0.28	0.05	0.09	0.09	0.09
Minneapolis	0.25	0.19	0.24	0.23	0.24	0.08	0.09	0.09	0.08
Paris	0.23	0.18	0.20	0.20	0.20	0.05	0.06	0.06	0.06

This table presents averages of predicted, implied and realised volatility of September wheat futures and options, where σ_{imp} denotes the average at the money implied volatility based on option with September futures as underlying, σ_{30d} denotes the mean predicted volatility based on historical 30 days average price volatility, σ_{imp}, σ_{ST} and $\sigma_{ST_{stu}}$ denote the average predicted volatility based on $Model-ST$ and conditioned version of $Model-ST$ using 25% as critical value fo the socks-to-use ratio, respectively.
Implied volatilities are taken from Bloomberg and log futures prices are taken from Thomson Reuters Datastream. The sample covers 9 marketing years between 2006/07 and 2016/17, where contracts are held from beginning of September to end of November. Daily realised volatility is performed from the forecast day onwards until the option's expiration. Forecast errors are reported as root mean squared errors (RSME) based on daily differences between model forecasts and realised volatility.

7.3 Summary

The findings provided in this section clearly show that historical 30 days average price volatility is an inferior predictor, which substantial underestimates realised volatility and produces larger RMSEs. Results are not unambiguous with respect to model comparison between implied volatility and the forecast of introduced deterministic volatility models. Both versions, $Model-ST$ and $Model-ST_{stu}$ provide superior mean volatility forecasts, while implied volatility provides smaller RMSEs, in particular for the two liquid option markets Chicago and Kansas. A remarkable result refers to Paris, where implied volatility overestimates mean realised volatility by almost 15% (3% points). This result indicates that the introduced deterministic volatility models have slightly superior forecast properties for less liquid option markets.

8 Conclusion

The first part of this study focuses on the explanation of observed implied volatility differences between September wheat options traded in Chicago and in Paris. Empirical findings in this study show that implied volatility coincide with historical volatility of the underlying September wheat futures contracts on both exchanges. This suggests, that volatility differences are driven by factors related to the market structure.

Three factors are identified helping to explain volatility differences: first, the USD/EUR exchange rate, quality and grading standards of wheat for delivery at the respective exchange, and market integration. Of primary importance is the USD/EUR exchange rate, where the inverse relationship between price and USD enlarges volatility when wheat contracts are valued in USD. Second, Chicago wheat is of minor quality compared to wheat allowed for delivery in Paris. Lower volatility differences are observed when quality spreads narrows, which is the case for Paris and Minneapolis wheat. Third, cancellation (introduction) of a market barrier, like a policy measure, can be regarded as a trigger point for strengthening market integration (segmentation). A stronger market foreclosure policy coincides with an increasing volatility spread between U.S. wheat and Paris wheat contracts.

This study provides empirical evidence that seasonality and time to maturity are both important elements for describing time varying volatility of wheat futures during a marketing year. Based on this findings a deterministic volatility model is introduced which disentangle seasonal and time to maturity effects. It further allows to incorporate storage information, as a fundamental driver of volatility and seasonality, which provides a profound basis for modelling large volatility spikes around harvest time. In particular, the seasonal amplitude depends on both, the volatility level and the storage level.

The introduced deterministic volatility model can be used for option pricing in the risk-neutral measure, where the option price is directly modelled on traded futures. This pricing approach can easily be applied in daily business and it also simplifies the valuation impact of extreme volatility shocks.

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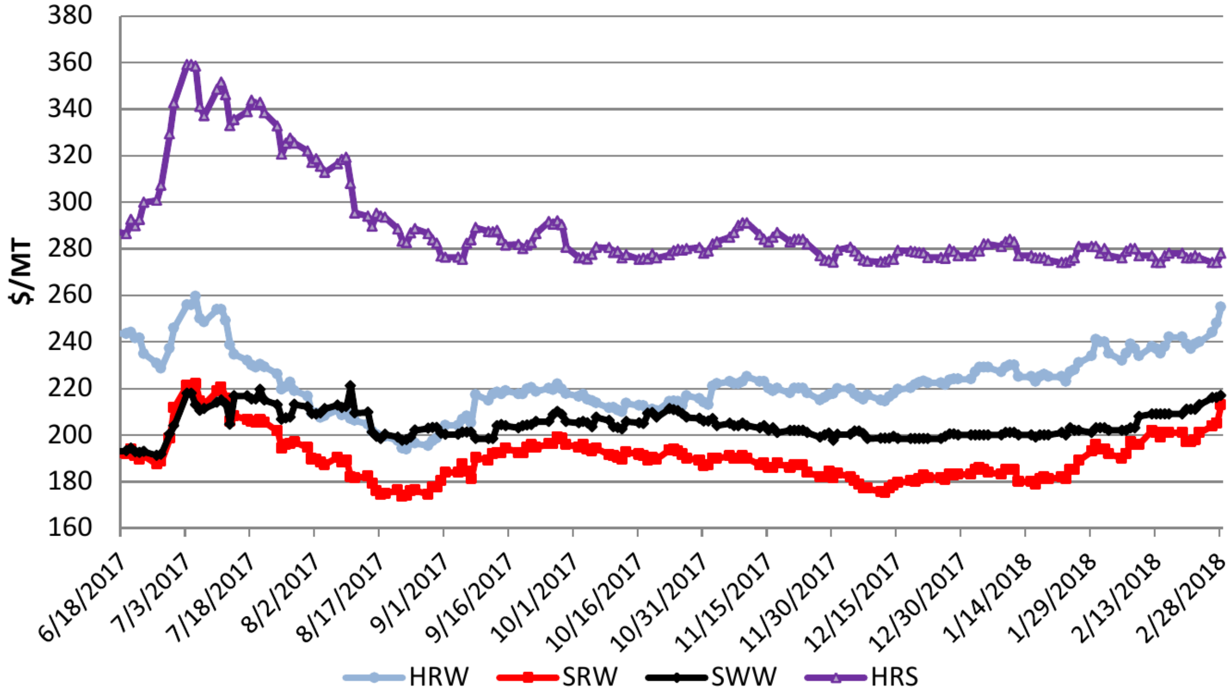
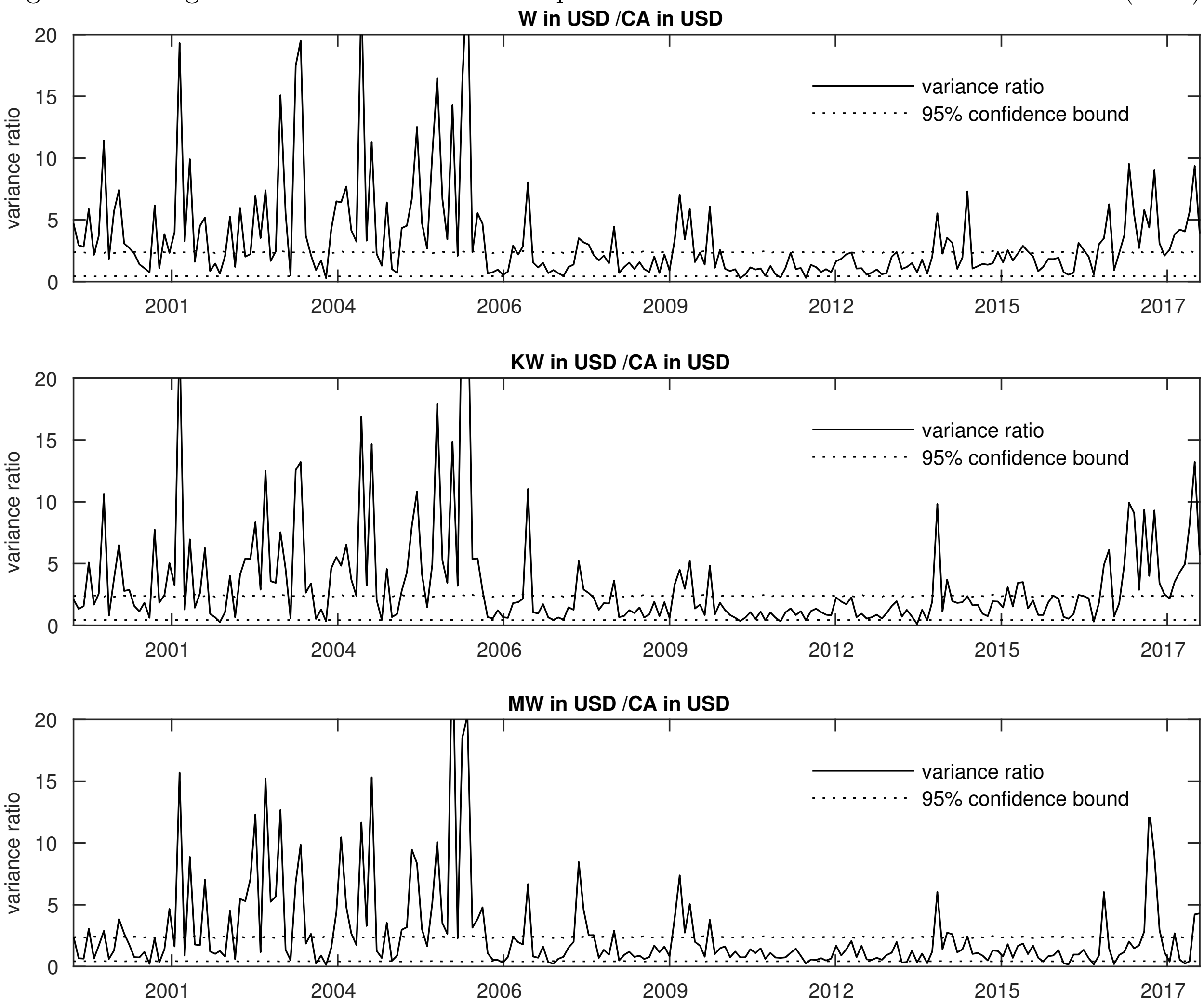
A Appendix

Figure 15: U.S. daily Free On Board (FOB) export bids for different wheat classes



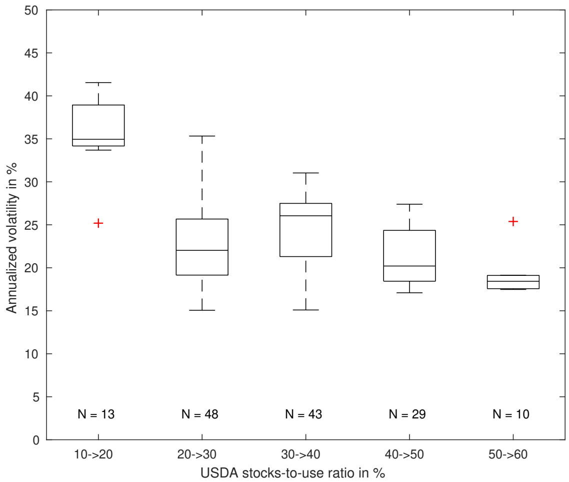
Source: Foreign Agricultural Service/USDA, "Wheat: world markets and trade" March 2018. Wheat classifications are: hard red winter wheat (HRW), soft red winter wheat (SRW), soft white wheat (SWW) and hard red spring wheat (HRS).

Figure 16: Range based variance F-test for September contracts between the U.S. and Paris (USD)

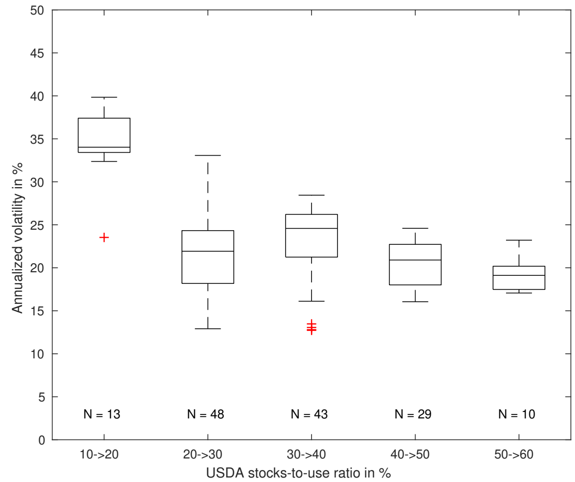


The figure shows the evolution of monthly variance ratios between U.S. wheat contracts and the Milling Wheat No. 2 contract traded at the EURONEXT (CA), converted into USD. All contracts expire in September, with the exception of Paris, where the November contract is used between 2008 and 2014. The contracts are held over one year form October to delivery in September. The range based variances are calculated using equation (1) and intra-month log high and low prices. W denotes the wheat contract traded at the CME, KW denotes the wheat contract traded at the KCBT and MW denotes the wheat contract traded at the MGEX.

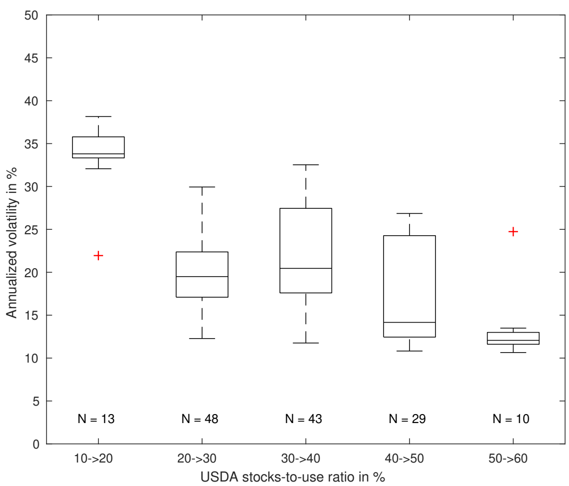
Figure 17: Impact stocks-to-use ratio on range based volatility



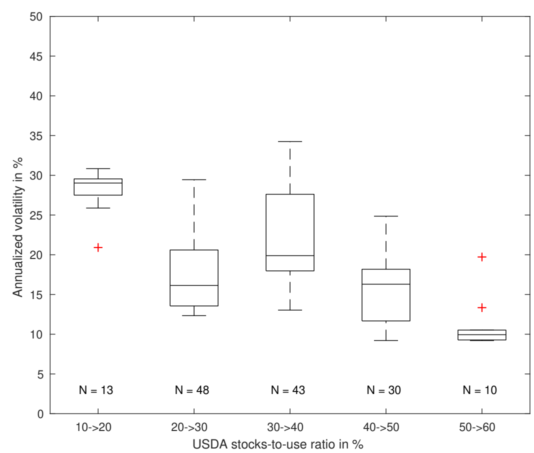
(a) Chicago Wheat (W), CME



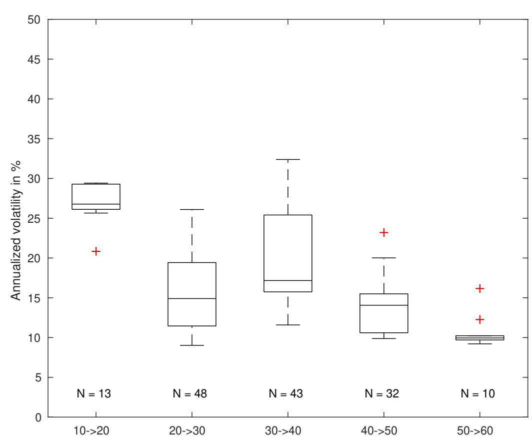
(b) Kansas Wheat (KW), KCBT



(c) Minneapolis Wheat (MW), MGEX



(d) Paris Milling Wheat (CA-USD), EURONEXT



(e) Paris Milling Wheat (CA-EUR), EURONEXT

WASDE U.S. wheat stocks-to-use ratios are taken form Blomberg covering the period January 2006 to December 2017 and are adjusted for the marketing year by postponing the ending stock projections from May to August. Monthly volatility levels are estimated using the robust version of Lowess as described in section 4, see figure 9.

Table 5: Wheat futures contract specifications and market characteristics

	Chicago SRW Wheat Futures	Paris Milling Wheat No. 2	Kansas HRW Wheat Futures	Minneapolis HRS Wheat Futures
Ticker Symbols	W	CA	KW	MW
Contract Unit	5.000 bushels (136 Metric Tonns)	50 Tonns	5.000 bushels (136 Metric Tonns)	5.000 bushels (136 Metric Tonns)
Deliverable Grade	No. 2 Soft Red Winter, No. 2 Hard Red Winter, No. 2 Dark Northern Spring and No. 2 Northern Spring at par. Other grades are acceptable for delivery at premiums and discounts.	Milling Wheat from any EU origin (soft wheat) protein levels of 11% dry matter since May 2017	Deliverable grades of HRW with a protein content of 11% or higher, with 11% to 10.5% deliverable at 10$ discount.	USDA No. 2 or better Dark Northern or Northern Spring Wheat with a protein content of 13.5% or higher, with 13.0% to 13.4% protein deliverable at a discount.
Delivery Location	Chicago area, Indiana area and the Toledo, Ohio area	Approved Silo in Rouen (France), and Dunkirk since 2015	Missouri (Kansas City) and Kansas (Hutchinson, Salina/Abilene and Wichita)	Minneapolis, Duluth and Minnesota regions
Price Quote	cents per bushel	cetns per tonne	cents per bushel	cents per bushel
Tick Size	0.25 $	0.25 €	0.25 $	0.25 $
Value	12.50 $	12.50 €	12.50 $	12.50 $
Contract Months	Jul, Sep, Dec, Mar, May	Nov, Jan, Mar, May until May 2015 Sep, Dec, Mar, May since Sep 2015	Jul, Sep, Dec, Mar, May	Jul, Sep, Dec, Mar, May
Last Trading Day	The business day prior to the 15th calendar day of the contract month.	18:30 on the tenth calendar day of the delivery month (if not a business day, then the following business day)	The business day prior to the 15th calendar day of the contract month.	The business day preceding the fifteenth calendar day of that contract month
Last Delivery Day	Second business day following the last trading day of the delivery month	Last business day of the delivery month.	Last business day of the delivery month.	The seventh business day following the last trading day
Trading Venue	CME Globex (Electronic Trading), TAS (Trade at Settlement)	Universal Trading Platform (UTP)	CME Globex (Electronic Trading), TAS (Trade at Settlement)	CME Globex (Electronic Trading),
Trading Hours	7:00 p.m. - 7:45 a.m. Sunday - Friday (CT) 8:30 a.m. - 1:20 p.m. Monday - Friday (CT)	10:45 – 18:30 CET	7:00 p.m. - 7:45 a.m. Sunday - Friday (CT) 8:30 a.m. - 1:20 p.m. Monday - Friday (CT)	7:00 p.m. - 7:45 a.m. Sunday - Friday (CT) 8:30 a.m. - 1:30 p.m. Monday - Friday (CT)
Orderbook	CLOB	COB	CLOB	CLOB

The table shows wheat futures contract specifications and market specifications for the commodity exchanges Chicago (CME), Paris (EURONEXT), Kansas (KCBT) and Minneapolis (MGEX)).

Table 6: Variance F-test: May contract U.S. vs. Paris wheat return based

season	Paris wheat in EUR			Paris wheat in USD		
	W	KW	MW	W	KW	MW
1999/00	11.94***	9.71***	6.50***	2.84***	2.31***	1.55***
2000/01	7.88***	6.15***	3.36***	1.60***	1.25**	0.68
2001/02	3.27***	2.50***	1.91***	1.75***	1.34***	1.02
2002/03	5.31***	4.54***	4.51***	2.72***	2.33***	2.31***
2003/04	3.43***	3.22***	2.90***	2.46***	2.31***	2.08***
2004/05	8.46***	6.38***	6.31***	3.75***	2.83***	2.80***
2005/06	9.96***	11.28***	7.47***	3.82***	4.32***	2.87***
2006/07	2.85***	2.14***	1.80***	2.44***	1.83***	1.54***
2007/08	1.56***	1.25**	1.30**	1.43***	1.14	1.19*
2008/09	3.23***	2.57***	2.06***	1.95***	1.56***	1.25**
2009/10	4.61***	3.85***	3.44***	2.52***	2.10***	1.88***
2010/11	1.39***	1.28**	1.11	1.22*	1.13	0.98
2011/12	1.46***	1.32**	0.92	1.18*	1.07	0.75
2012/13	1.83***	1.89***	1.24**	1.55***	1.60***	1.05
2013/14	2.23***	1.84***	1.24**	1.90***	1.58***	1.06
2014/15	1.98***	1.79***	1.35***	1.56***	1.41***	1.07
2015/16	2.36***	2.26***	1.35***	2.00***	1.91***	1.14
2016/17	3.27***	3.29***	1.53***	2.54***	2.55***	1.19*
2017/18	4.32***	4.98***	3.15***	4.12***	4.75***	3.01***

Season stands for the observed marketing year, which starts in June and ends in May of the following year ensuring that one marketing year of particular contract is covered. W refers to the wheat contract traded on the CME, KW is traded on the KCBT and MW is traded on the MGEX. Variances are calculated using daily log returns over the contract's marketing year. The table gives the one side F-value test statistics, i.e variance ratios between a May U.S. wheat contract and a May Paris wheat contract, implying a rejection of the null hypothesis of equal variances at the 10%/5%/1% level of significance (marked with $*/**/***$).

Table 7: Variance F-test: September contract U.S. vs. Paris wheat range based

season	Paris wheat in EUR			Paris wheat in USD		
	W	KW	MW	W	KW	MW
1999/00	27.64***	12.41***	18.59***	2.21***	0.99	1.49***
2000/01	5.05***	4.22***	3.73***	2.43***	2.03***	1.79***
2001/02	2.71***	4.98***	5.08***	3.06***	5.61***	5.73***
2002/03	2.36***	2.24***	1.53***	1.17*	1.11	0.76
2003/04	2.91***	2.30***	2.15***	2.98***	2.34***	2.19***
2004/05	2.20***	1.36***	1.36***	1.39***	0.86	0.86
2005/06	1.15	2.46***	3.02***	0.77	1.65***	2.02***
2006/07	0.59	0.42	0.30	0.50	0.35	0.26
2007/08	5.02***	5.04***	6.08***	4.00***	4.01***	4.84***
2008/09	2.24***	1.89***	1.75***	1.53***	1.29**	1.19*
2009/10	0.87	0.71	0.70	0.81	0.66	0.65
2010/11	1.32**	0.85	1.03	1.08	0.70	0.85
2011/12	0.87	0.77	0.59	1.20*	1.07	0.81
2012/13	1.35***	1.02	0.91	1.66***	1.25**	1.11
2013/14	2.57***	2.33***	1.79***	2.01***	1.82***	1.40***
2014/15	1.85***	2.87***	1.75***	1.09	1.69***	1.03
2015/16	2.71***	2.76***	0.54	2.52***	2.57***	0.50
2016/17	3.71***	3.96***	6.85***	5.29***	5.65***	9.78***
2017/18	2.32***	3.75***	0.69	5.42***	8.78***	1.61***

Season stands for the observed marketing year, which starts in October and ends in September of the following year ensuring that one marketing year of particular contract is covered. W refers to the wheat contract traded on the CME, KW is traded on the KCBT and MW is traded on the MGEX. Variances are calculated using equation (1) and intra-year log high and low prices over the contract's marketing year, i.e. the obs. year. The table gives the one side F-value test statistics, i.e variance ratios between a September U.S. wheat contract and a September Paris wheat contract, implying a rejection of the null hypothesis of equal variances at the 10%/5%/1% level of significance (marked with $*/**/***$).